Data-Flow Analysis

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TU Delft

CS4200 | Compiler Construction | January 7, 2021

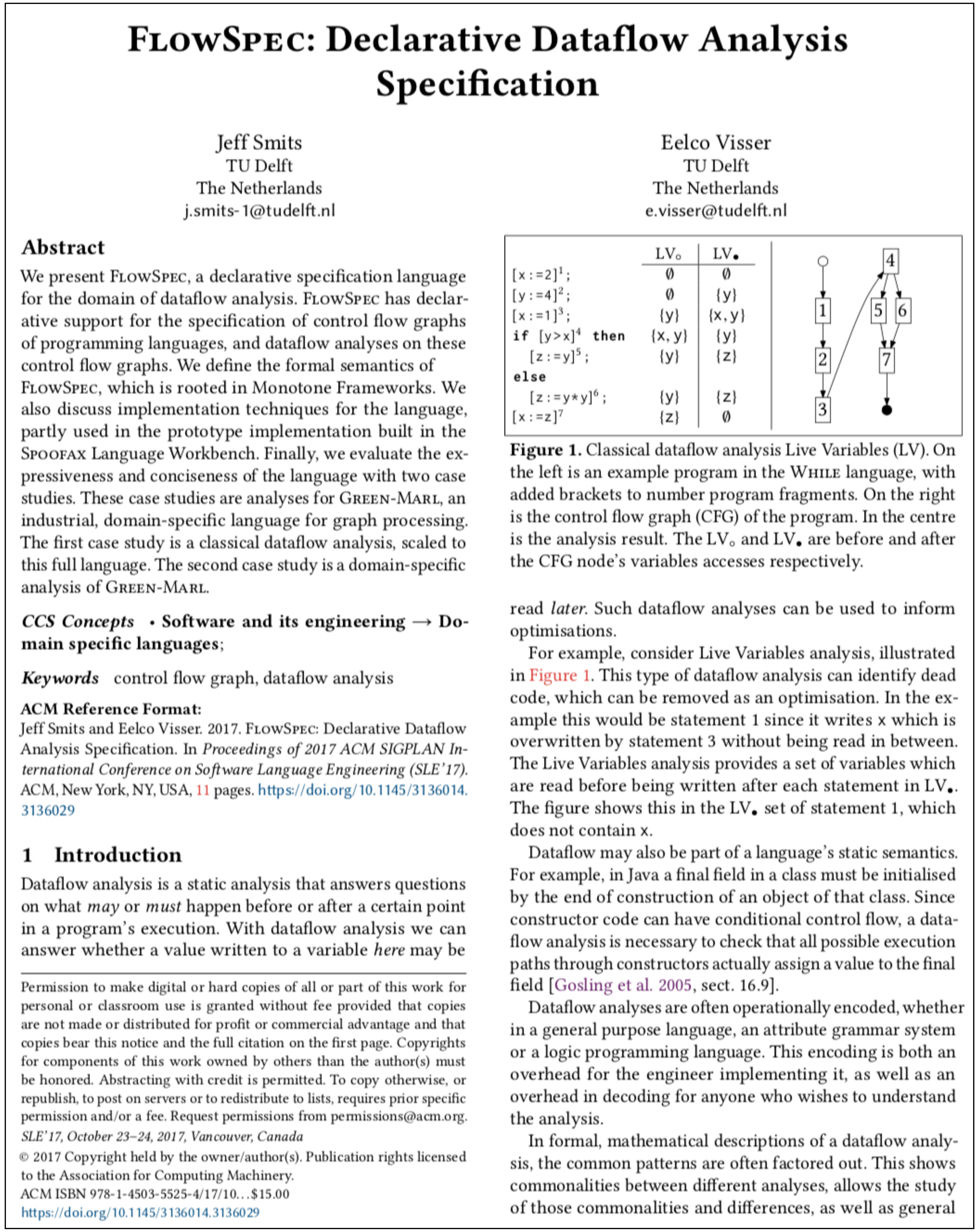
Reading Material

The following papers add background, conceptual exposition, and examples to the material from the slides. Some notation and technical details have been changed; check the documentation.

This paper introduces FlowSpec, the declarative data-flow analysis specification language in Spoofax. Although the design of the language described in this paper is still current, the syntax used is already dated, i.e. the current FlowSpec syntax in Spoofax is slightly different.

SLE 2017

https://doi.org/10.1145/3136014.3136029



FlowSpec: Declarative Dataflow Analysis Specification

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Abstract

We present FlowSpec, a declarative specification language for the domain of dataflow analysis. FlowSpec has declarative support for the specification of control flow graphs of programming languages, and dataflow analyses on these control flow graphs. We define the formal semantics of FlowSpec, which is rooted in Monotone Frameworks. We also discuss implementation techniques for the language, partly used in the prototype implementation built in the Spoofax Language Workbench. Finally, we evaluate the expressiveness and conciseness of the language with two case studies. These case studies are analyses for Green-Marl, an industrial, domain-specific language for graph processing. The first case study is a classical dataflow analysis, scaled to this full language. The second case study is a domain-specific analysis of Green-Marl.

CCS Concepts • **Software and its engineering → Domain specific languages;**

Keywords control flow graph, dataflow analysis

ACM Reference Format:
Jeff Smits and Eelco Visser. 2017. FlowSpec: Declarative Dataflow Analysis Specification. In *Proceedings of 2017 ACM SIGPLAN International Conference on Software Language Engineering (SLE'17).* ACM, New York, NY, USA, 11 pages. https://doi.org/10.1145/3136014.3136029

1 Introduction

Dataflow analysis is a static analysis that answers questions on what *may* or *must* happen before or after a certain point in a program's execution. With dataflow analysis we can answer whether a value written to a variable *here* may be read *later*. Such dataflow analyses can be used to inform optimisations.

	$LV_\circ$	$LV_\bullet$
`[x:=2]`^{1};	∅	∅
`[y:=4]`^{2};	∅	{y}
`[x:=1]`^{3};	{y}	{x, y}
if `[y>x]`^{4} **then**	{x, y}	{y}
`[z:=y]`^{5};	{y}	{z}
else		
`[z:=y*y]`^{6};	{y}	{z}
`[x:=z]`^{7}	{z}	∅

Figure 1. Classical dataflow analysis Live Variables (LV). On the left is an example program in the While language, with added brackets to number program fragments. On the right is the control flow graph (CFG) of the program. In the centre is the analysis result. The $LV_\circ$ and $LV_\bullet$ are before and after the CFG node's variables accesses respectively.

For example, consider Live Variables analysis, illustrated in Figure 1. This type of dataflow analysis can identify dead code, which can be removed as an optimisation. In the example this would be statement 1 since it writes x which is overwritten by statement 3 without being read in between. The Live Variables analysis provides a set of variables which are read before being written after each statement in $LV_\bullet$. The figure shows this in the $LV_\bullet$ set of statement 1, which does not contain x.

Dataflow may also be part of a language's static semantics. For example, in Java a final field in a class must be initialised by the end of construction of an object of that class. Since constructor code can have conditional control flow, a dataflow analysis is necessary to check that all possible execution paths through constructors actually assign a value to the final field [Gosling et al. 2005, sect. 16.9].

Dataflow analyses are often operationally encoded, whether in a general purpose language, an attribute grammar system or a logic programming language. This encoding is both an overhead for the engineer implementing it, as well as an overhead in decoding for anyone who wishes to understand the analysis.

In formal, mathematical descriptions of a dataflow analysis, the common patterns are often factored out. This shows commonalities between different analyses, allows the study of those commonalities and differences, as well as general

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Journal version of the SLE paper.

This paper introduces FlowSpec, the declarative data-flow analysis specification language in Spoofax.

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FLOWSPEC: A declarative specification language for intra-procedural flow-Sensitive data-flow analysis

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HIGHLIGHTS

- Data-flow analysis is the static analysis of programs to estimate their approximate run-time behavior or approximate intermediate run-time values. It is an integral part of modern language specifications and compilers. In the specification of static semantics of programming languages, the concept of data-flow allows the description of well-formedness such as definite assignment of a local variable before its first use. In the implementation of compiler back-ends, data-flow analyses inform optimizations.
- Data-flow analysis has an established theoretical foundation. What lags behind is implementations of data-flow analysis in compilers, which are usually ad-hoc. This makes such implementations difficult to extend and maintain. In previous work researchers have proposed higher-level formalisms suitable for whole-program analysis in a separate tool, incremental analysis within editors, or bound to a specific intermediate representation.
- In this paper, we present FlowSpec, an executable formalism for specification of data-flow analysis. FlowSpec is a domain-specific language that enables direct and concise specification of data-flow analysis for programming languages, designed to express flow-sensitive, intra-procedural analyses.
- We define the formal semantics of FlowSpec in terms of monotone frameworks. We describe the design of FlowSpec using examples of standard analyses. We also include a description of our implementation of FlowSpec.
- In a case study we evaluate FlowSpec with the static analyses for GreenMarl, a domain-specific programming language for graph analytics.

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ABSTRACT

Data-flow analysis is the static analysis of programs to estimate their approximate run-time behavior or approximate intermediate run-time values. It is an integral part of modern language specifications and compilers. In the specification of static semantics of programming languages, the concept of data-flow allows the description of well-formedness such as definite assignment of a local variable before its first use. In the implementation of compiler back-ends, data-flow analyses inform optimizations.

Data-flow analysis has an established theoretical foundation. What lags behind is implementations of data-flow analysis in compilers, which are usually ad-hoc. This makes such implementations difficult to extend and maintain. In previous work researchers have proposed higher-level formalisms suitable for whole-program analysis in a separate tool, incremental analysis within editors, or bound to a specific intermediate representation.

In this paper, we present FLOWSPEC, an executable formalism for specification of data-flow analysis. FLOWSPEC is a domain-specific language that enables direct and concise specification of data-flow analysis for programming languages, designed to express flow-sensitive, intra-procedural analyses. We define the formal semantics of FLOWSPEC in terms of monotone frameworks. We describe the design of FLOWSPEC using examples of standard analyses. We also include a description of our implementation of FLOWSPEC.

In a case study we evaluate FLOWSPEC with the static analyses for GREEN-MARL, a domain-specific programming language for graph analytics.

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Documentation for FlowSpec at the metaborg.org website.

Spoofax

latest

Search docs

The Spoofax Language Workbench
Examples
Publications

TUTORIALS

Installing Spoofax
Creating a Language Project
Using the API
Getting Support

REFERENCE MANUAL

Language Definition with Spoofax
Abstract Syntax with ATerms
Syntax Definition with SDF3
Static Semantics with NaBL2
Data-Flow Analysis with FlowSpec
1. Introduction
2. Language Reference
3. Stratego API
4. Configuration
5. Examples (under construction)
6. Bibliography
Transformation with Stratego
Dynamic Semantics with DynSem
Editor Services with ESV
Language Testing with SPT
...Command-line

Read the Docs v: latest

Docs » Data Flow Analysis Definition with FlowSpec

Edit on GitHub

Data Flow Analysis Definition with FlowSpec

Programs that are syntactically well-formed are not necessarily valid programs. Programming languages typically impose additional *context-sensitive* requirements on programs that cannot be captured in a syntax definition. Languages use data and control flow to check certain extra properties that fall outside of names and type systems. The FlowSpec 'Flow Analysis Specification Language' supports the specification of rules to define the static control flow of a language, and data flow analysis over that control flow. FlowSpec supports flow-sensitive intra-procedural data flow analysis.

Table of Contents

http://www.metaborg.org/en/latest/source/langdev/meta/lang/flowspec/index.html

Data-Flow Analysis

What is Data-Flow Analysis?

What is Data-Flow Analysis?

Static approximation of runtime behaviour

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed

Available Expressions

```
let
  var x : int := a + b
  var y : int := a * b
 in
  while y > a + b then
    (
      a := a + 1;
      x := a + b
    )
end
```

Available Expressions

```
let
   var x : int := a + b
   var y : int := a * b
 in
   while y > a + b then
     (
       a := a + 1;
       x := a + b
     )
end
```

- a + b is *already computed* when you get to the condition
- There is no need to compute it again

Live Variables

```
x := 2;
y := 4;
x := 1;
if y > x then
  z := y
else
  z := y * y;
x := z
```

Live Variables

```
x := 2;
y := 4;
x := 1;
if y > x then
  z := y
else
  z := y * y;
x := z
```

The first value of x is never observed,
because it isn't read after the assignment

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed
- What extra invariants do some data adhere to

Flow-Sensitive Types

```
void hello(String? name) {
    if (is String name) {
        // name is of type String here
        print("Hello, ``name``!");
    }
    else {
        print("Hello, world!");
    }
}
```

Flow-Sensitive Types

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    }
}
```

- Ceylon (https://ceylon-lang.org/)
- Union and intersection types
- **String?** ≡ String | Null
- **is** like Java's **instanceof**
- General name: path-sensitive data-flow analysis

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed
- What extra invariants do some data adhere to

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed
- What extra invariants do some data adhere to
- Data dependence between data/variables where the data lives

Reaching Definitions

```
let
  var x : int := 5
  var y : int := 1
 in
  while x > 1 do
    (
      y := x * 2;
      x := y - 1
    )
end
```

- The inverse relation of live variables
- RD gives us the possible origins of the current value of a variable

Reaching Definitions

```
let
  var x : int := 5      x↦2
  var y : int := 1      x↦2;y↦3
 in
  while x > 1 do        x↦2,8;y↦3,7
    (
      y := x * 2;       x↦2,8;y↦7
      x := y - 1        x↦8;y↦7
    )
end
```

- Analysis result is a multi-map (shown here after each statement)
- Propagate information along the control-flow graph

Reaching Definitions

```
1  let
2    var x : int := 5 ________ x↦2             {(x,2)}
3    var y : int := 1 ________ x↦2;y↦3         {(x,2) (y,3)}
4   in
5    while x > 1 do ________ x↦2,8;y↦3,7       {(x,2) (x,8) (y,3) (y,7)}
6      (
7        y := x * 2; ________ x↦2,8;y↦7        {(x,2) (x,8) (y,7)}
8        x := y - 1 ________ x↦8;y↦7           {(x,8) (y,7)}
9      )
10 end
```

- Analysis result is a *set of pairs* (shown here after each statement)
- Propagate information along the control-flow graph

Control-Flow

Control-Flow

What is Control-Flow?

Control-Flow

What is Control-Flow?

– "Order of evaluation"

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– "Order of evaluation"

Discuss a series of example programs

Control-Flow

What is Control-Flow?

– “Order of evaluation”

Discuss a series of example programs

– What is the control flow?

What is Control-Flow?

- “Order of evaluation”

Discuss a series of example programs

- What is the control flow?
- What constructs in the program determine that?

What is Control-Flow?

```
function id(x) { return x; }
id(4); id(true);
```

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function id(x) { return x; }
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Function calls

What is Control-Flow?

```
function id(x) { return x; }
id(4); id(true);
```

Function calls

- Calling a function passes control to that function
- A **return** passes control back to the caller

What is Control-Flow?

```
if (c) { a = 5 } else { a = "four" }
```

What is Control-Flow?

```
if (c) { a = 5 } else { a = "four" }
```

Branching

What is Control-Flow?

if (c) { a = 5 } **else** { a = "four" }

Branching

- Control is passed to one of the two branches
- This is dependent on the value of the condition

What is Control-Flow?

```
while (c) { a = 5 }
```

What is Control-Flow?

while (c) { a = 5 }

Looping

What is Control-Flow?

while (c) { a = 5 }

Looping

- Control is passed to the loop body depending on the condition
- After the body we start over

What is Control-Flow?

```
function1(a);
function2(b);
```

What is Control-Flow?

```
function1(a);
function2(b);
```

Sequence

What is Control-Flow?

```
function1(a);
function2(b);
```

Sequence

- No conditions or anything complicated
- But still order of execution

What is Control-Flow?

```
distance = distance + 1;
```

What is Control-Flow?

```
distance = distance + 1;
```

Reads and Writes

What is Control-Flow?

```
distance = distance + 1;
```

Reads and Writes

- The expression needs to be evaluated, before we can save its result

What is Control-Flow?

```
int i = 2;
int j = (i=3) * i;
```

What is Control-Flow?

```
int i = 2;
int j = (i=3) * i;
```

Expressions & side-effects

What is Control-Flow?

```
int i = 2;
int j = (i=3) * i;
```

Expressions & side-effects

- Order in sub-expressions is usually undefined
- Side-effects make sub-expression order relevant

Kinds of Control-Flow

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- Sequential statements

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- Sequential	statements
- Conditional	if / switch / case

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- Async-await	threading

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- Coroutines / Generators	yield

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- Dispatch	function calls / method calls

Kinds of Control-Flow

- Sequential	statements
- Conditional	if / switch / case
- Looping	while / do while / for / foreach / loop
- Exceptions	throw / try / catch / finally
- Continuations	call/cc
- Async-await	threading
- Coroutines / Generators	yield
- Dispatch	function calls / method calls
- Loop jumps	break / continue
- ... many more ...	

Why Control-Flow?

Why Control-Flow?

Shorter code

- No need to repeat the same statement 10 times

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Parametric code

- Extract reusable patterns
- Let user decide repetition amount

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Expressive power

- Playing with Turing Machines

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Reason about program execution

- What happens when?
- In what order?

Control-Flow and Language Design

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Imperative programming

- Explicit control-flow constructs

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Declarative programming

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- What, not how
- Less explicit control-flow
- More options for compilers to choose order

Control-Flow and Language Design

Imperative programming

- Explicit control-flow constructs

Declarative programming

- What, not how
- Less explicit control-flow
- More options for compilers to choose order
- Great if your compiler is often smarter than the programmer

Separation of Concerns in Data-Flow Analysis

Separation of Concerns in Data-Flow Analysis

Representation

- Represent control-flow of a program

Separation of Concerns in Data-Flow Analysis

Representation

- Represent control-flow of a program
- Conduct and represent results of data-flow analysis

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Declarative Rules

- To define control-flow of a language
- To define data-flow of a language

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Declarative Rules

- To define control-flow of a language
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Language-Independent Tooling

- Data-Flow Analysis
- Errors/Warnings
- Code completion
- Refactoring
- Optimisation
- …

Control-Flow Graphs

What is a Control-Flow Graph?

A **control flow graph** (**CFG**) in computer science is a representation, using graph notation, of all paths that might be traversed through a program during its execution.

https://en.wikipedia.org/wiki/Control_flow_graph

Control-Flow Graph Example

```
let
  var x : int := a + b
  var y : int := a * b
 in
  while y > a + b do
    (
      a := a + 1;
      x := a + b
    )
end
```

Control-Flow Graph Example

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let
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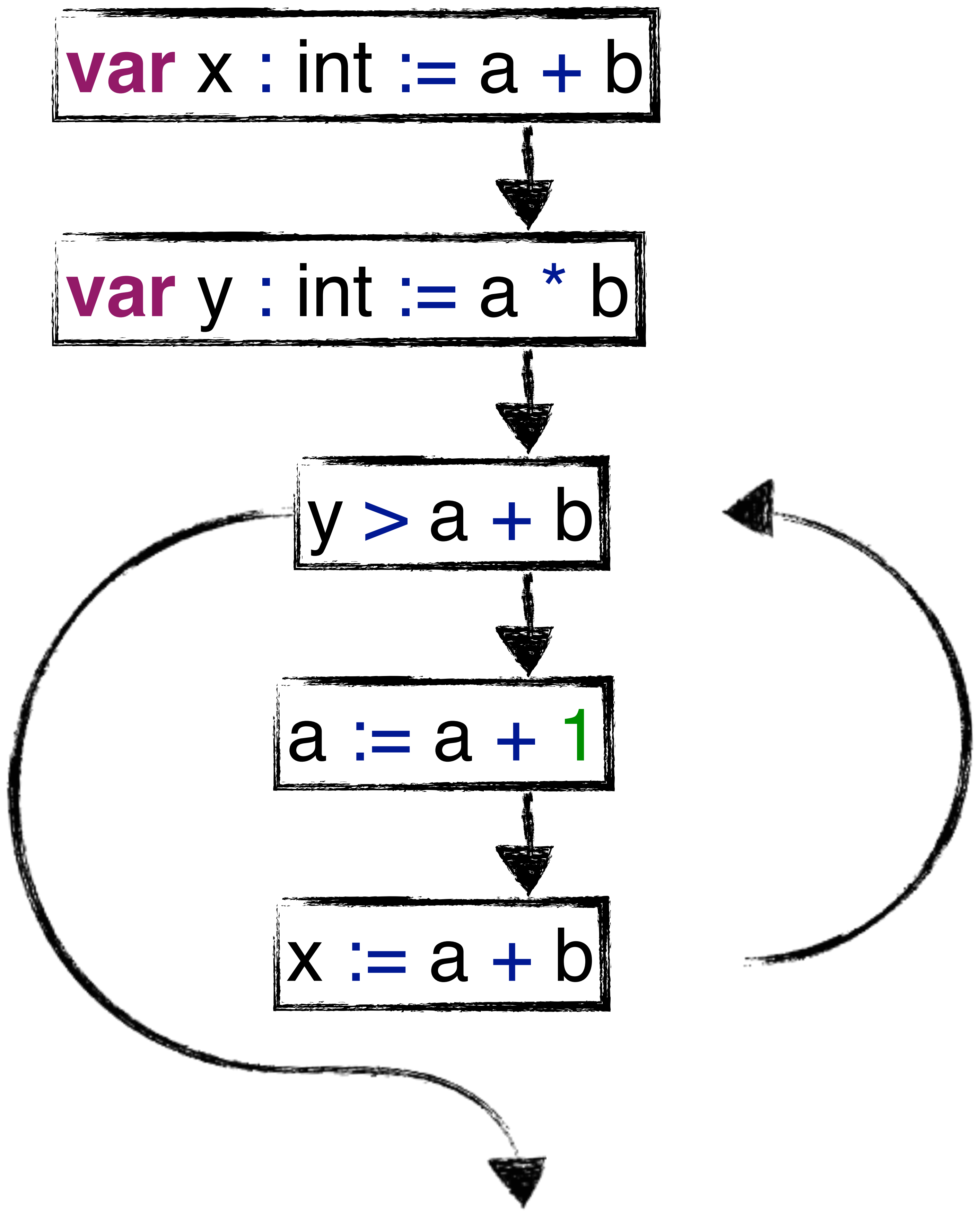

Basic Blocks

```
let
  var x : int := a + b
  var y : int := a * b
 in
  while y > a + b do
    (
      a := a + 1;
      x := a + b
    )
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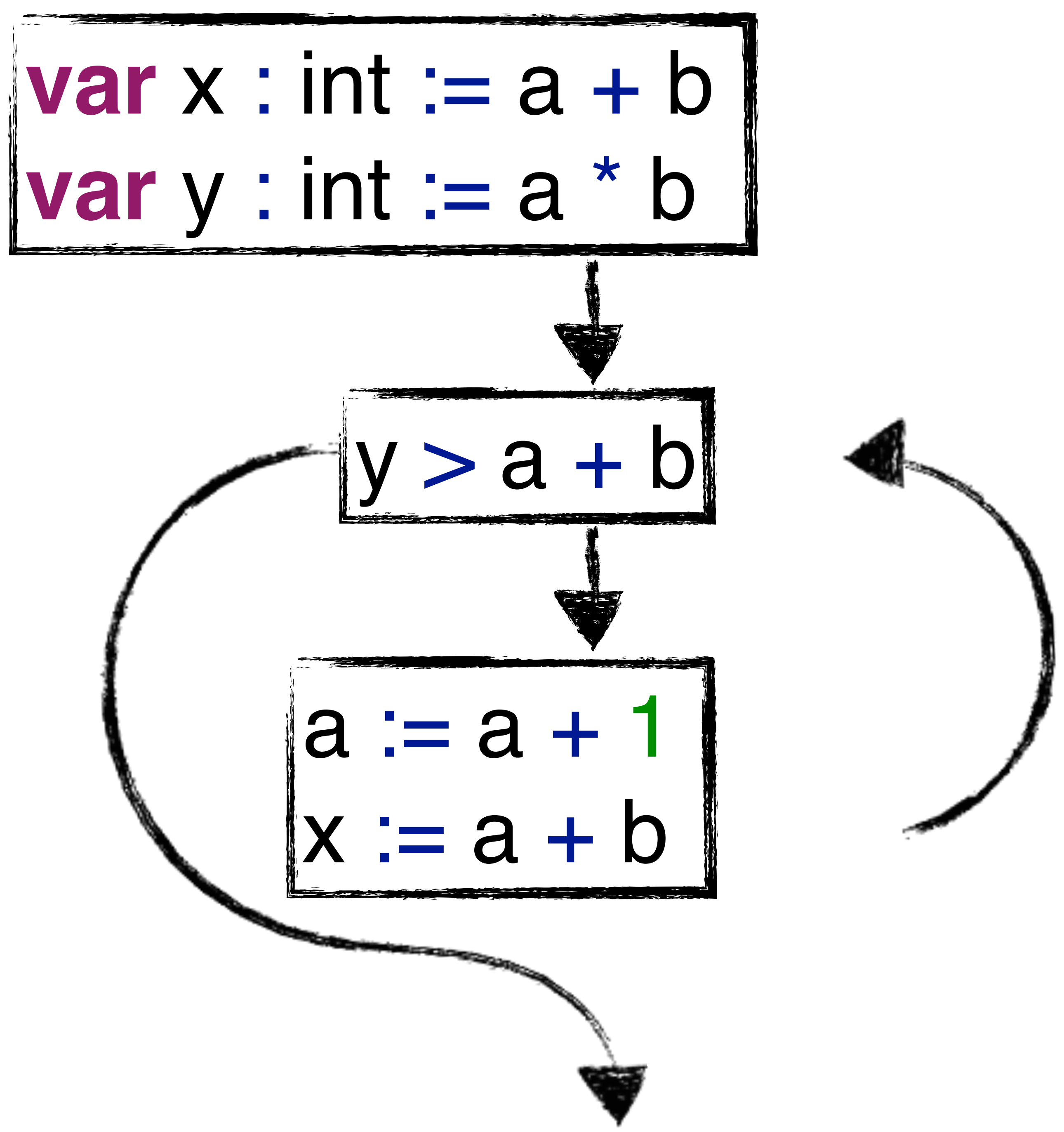

Control Flow Graphs

Control Flow Graphs

Nodes

- Usually innermost statements and expressions
- Or blocks for consecutive statements (basic blocks)

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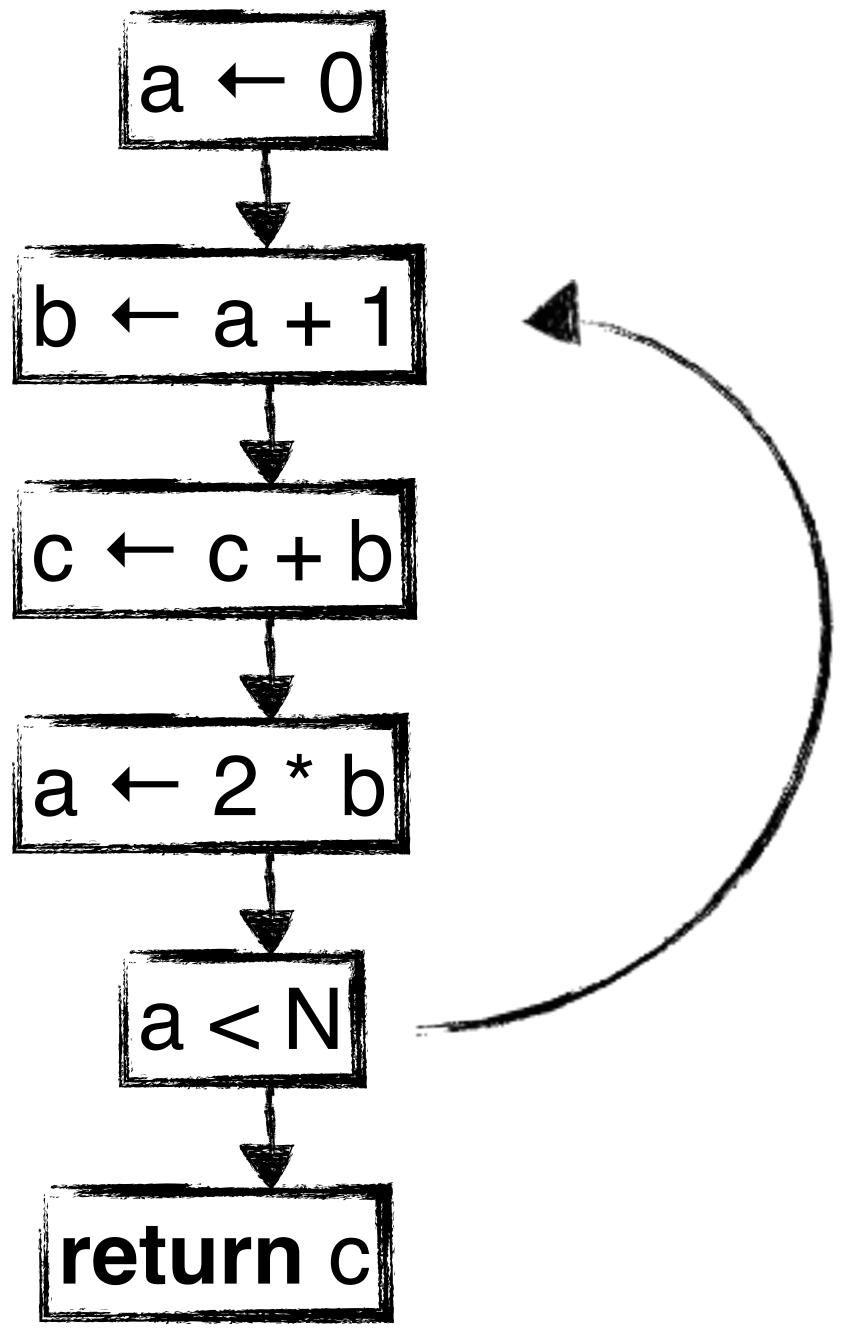
- Or blocks for consecutive statements (basic blocks)

Edges

- Back edges: show loops
- Splits: conditionally split the control flow
- Merges: combine previously split control flow

Equivalent to Unstructured Control-Flow

```
    a ← 0
L1: b ← a + 1
    c ← c + b
    a ← 2 * b
    if a < N goto L1
    return c
```



Separation of Concerns in Data-Flow Analysis

Representation

- Represent control-flow of a program
- Conduct and represent results of data-flow analysis

Declarative Rules

- To define control-flow of a language
- To define data-flow of a language

Language-Independent Tooling

- Data-Flow Analysis
- Errors/Warnings
- Code completion
- Refactoring
- Optimisation
- …

Separation of Concerns in Data-Flow Analysis

Representation

- Control Flow Graphs (CFGs)
- Conduct and represent results of data-flow analysis

Declarative Rules

- To define control-flow of a language
- To define data-flow of a language

Language-Independent Tooling

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Data-Flow

Data-Flow

What is Data-Flow?

What is Data-Flow?

- Possible values (data) that flow through the program

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- Possible values (data) that flow through the program
- Relations between those data (data dependence)

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Discuss a series of example programs

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- What is the flow we can use for this?

Data-Flow

What is Data-Flow?

- Possible values (data) that flow through the program
- Relations between those data (data dependence)

Discuss a series of example programs

- What is wrong or can be optimised?
- What is the flow we can use for this?
- What would the data-flow information look like?

What is wrong here?

```
public int ComputeFac(int num) {
    return num;
    int num_aux;
    if (num < 1)
        num_aux = 1;
    else
        num_aux = num * this.ComputeFac(num-1);
    return num_aux;
}
```

What is wrong here?

```
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Dead code (control-flow)

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        num_aux = num * this.ComputeFac(num-1);
    return num_aux;
}
```

Dead code (control-flow)

- Most of the code is never reached because of the early return
- This is usually considered an error by compilers

What is “wrong” here?

```
x := 2;
y := 4;
x := 1;
// x and y used later
```

What is “wrong” here?

```
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Dead code (data-flow)

What is "wrong" here?

```
x := 2;
y := 4;
x := 1;
// x and y used later
```

Dead code (data-flow)

- The first value of x is never observed
- This is sometimes warned about by compilers

What is "wrong" here?

```
x := 2;
y := 4;
x := 1;
// x and y used later
```

Dead code (data-flow)

Live variable analysis

- The first value of x is never observed
- This is sometimes warned about by compilers

What is suboptimal here?

```
let
  var x : int := a + b
  var y : int := a * b
 in
  if y > a + b then
    (
      a := a + 1;
      x := a + b
    )
end
```

What is suboptimal here?

```
let
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      a := a + 1;
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end
```

Common subexpression elimination

What is suboptimal here?

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 in
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end
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Common subexpression elimination

- a + b is already computed when you get to the condition
- There is no need to compute it again

What is suboptimal here?

```
let
   var x : int := a + b
   var y : int := a * b
 in
   if y > a + b then
     (
        a := a + 1;
        x := a + b
     )
end
```

Common subexpression elimination

Available expression analysis

- a + b is already computed when you get to the condition
- There is no need to compute it again

What is suboptimal here?

```
for i := 1 to 100 do
  (
    x[i] := y[i];
    if w > 0 then
      y[i] := 0
  )
```

What is suboptimal here?

```
for i := 1 to 100 do
  (
    x[i] := y[i];
    if w > 0 then
      y[i] := 0
  )
```

Loop unswitching

What is suboptimal here?

```
for i := 1 to 100 do
  (
    x[i] := y[i];
    if w > 0 then
      y[i] := 0
  )
```

Loop unswitching

- The if condition is not dependent on i, x or y
- Still it is checked in the loop, which is slowing the loop

What is suboptimal here?

```
for i := 1 to 100 do
  (
    x[i] := y[i];
    if w > 0 then
      y[i] := 0
  )
```

Loop unswitching

Data-dependence analysis

- The if condition is not dependent on i, x or y
- Still it is checked in the loop, which is slowing the loop

Separation of Concerns in Data-Flow Analysis

Representation

- Control Flow Graphs (CFGs)
- Conduct and represent results of data-flow analysis

Declarative Rules

- To define control-flow of a language
- To define data-flow of a language

Language-Independent Tooling

- Data-Flow Analysis
- Errors/Warnings
- Code completion
- Refactoring
- Optimisation
- …

Separation of Concerns in Data-Flow Analysis

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- Control Flow Graphs (CFGs)
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Declarative Rules

- A domain-specific meta-language for Spoofax: FlowSpec

Language-Independent Tooling

- Data-Flow Analysis
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- ...

Tiger in FlowSpec

Control-Flow Rules

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

- Match an AST pattern

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

- Match an AST pattern
- List all CFG edges of that AST

Control-Flow Rules

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- Mark subtrees as CFG nodes

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

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- List all CFG edges of that AST
- Mark subtrees as CFG nodes
- Or splice in their control-flow subgraph

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

- Match an AST pattern
- List all CFG edges of that AST
- Mark subtrees as CFG nodes
- Or splice in their control-flow subgraph
- Use special “context” nodes to connect subgraph to outside graph

Control-Flow Graphs in FlowSpec

FlowSpec

Example program

```
x := 1;
if y > x then
  z := y;
else
  z := y * y;
y := a * b;
while y > a + b do
 (a := a + 1;
  x := a + b)
```

Control-Flow Graphs in FlowSpec

FlowSpec

```
root Mod(s) =
  start → s,
  s → end
```

Example program

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x := 1;
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end

Control-Flow Graphs in FlowSpec

FlowSpec

```
root Mod(s) =
  start → s → end

a@Assign(_, _) =
  entry → node a → exit
```

Example program

start

```
x := 1;
if y > x then
  z := y;
else
  z := y * y;
y := a * b;
while y > a + b do
 (a := a + 1;
  x := a + b)
```

end

Control-Flow Graphs in FlowSpec

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Example program

```
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if y > x then

  z := y;

else

  z := y * y;

y := a * b;

while y > a + b do

  (a := a + 1;

  x := a + b)

end
```

Control-Flow Graphs in FlowSpec

FlowSpec

```
root Mod(s) =
  start → s → end

Assign(_, _) =
  entry → this → exit
```

Example program

```
start

x := 1;

if y > x then
  z := y;
else
  z := y * y;
y := a * b;
while y > a + b do
  (a := a + 1;
  x := a + b)

end
```

Control-Flow Graphs in FlowSpec

FlowSpec

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root Mod(s) =
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node Assign(_, _)
```

Example program

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Control-Flow Graphs in FlowSpec

FlowSpec

```
root Mod(s) =
  start → s → end

node Assign(_, _)


Seq(s1, s2) =
  entry → s1 → s2 → exit
```

Example program

```
start

x := 1;

if y > x then
  z := y;
else
  z := y * y;
y := a * b;
while y > a + b do
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end
```

Control-Flow Graphs in FlowSpec

FlowSpec

```
root Mod(s) =
  start → s → end


node Assign(_, _)


Seq(s1, s2) =
  entry → s1 → s2 → exit

IfThenElse(c, t, e) =
  entry → node c → t → exit,
          node c → e → exit
```

Example program

```
start
x := 1;
if y > x then
  z := y;
else
  z := y * y;
y := a * b;
while y > a + b do
  (a := a + 1;
  x := a + b)
end
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Control-Flow Graphs in FlowSpec

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  entry → node c → t → exit,
          node c → e → exit
```

Example program

```
start
x := 1;
if y > x then
  z := y;
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  z := y * y;
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while y > a + b do
  (a := a + 1;
  x := a + b)
end
```

Control-Flow Graphs in FlowSpec

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Example program

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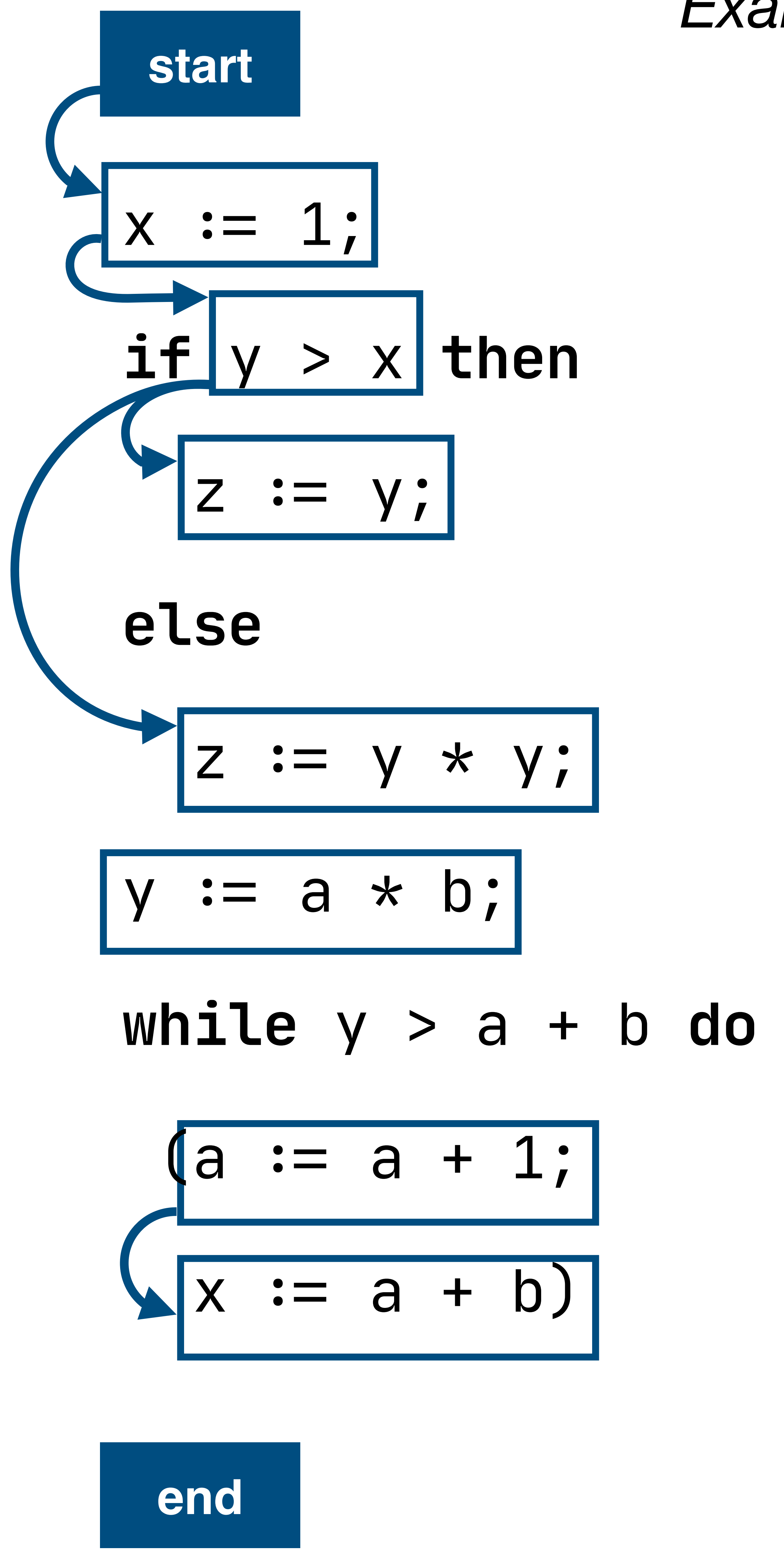
Example program



Control-Flow Graphs in FlowSpec

FlowSpec

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root Mod(s) =
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node Assign(_, _)


Seq(s1, s2) =
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IfThenElse(c, t, e) =
  entry → node c → t → exit,
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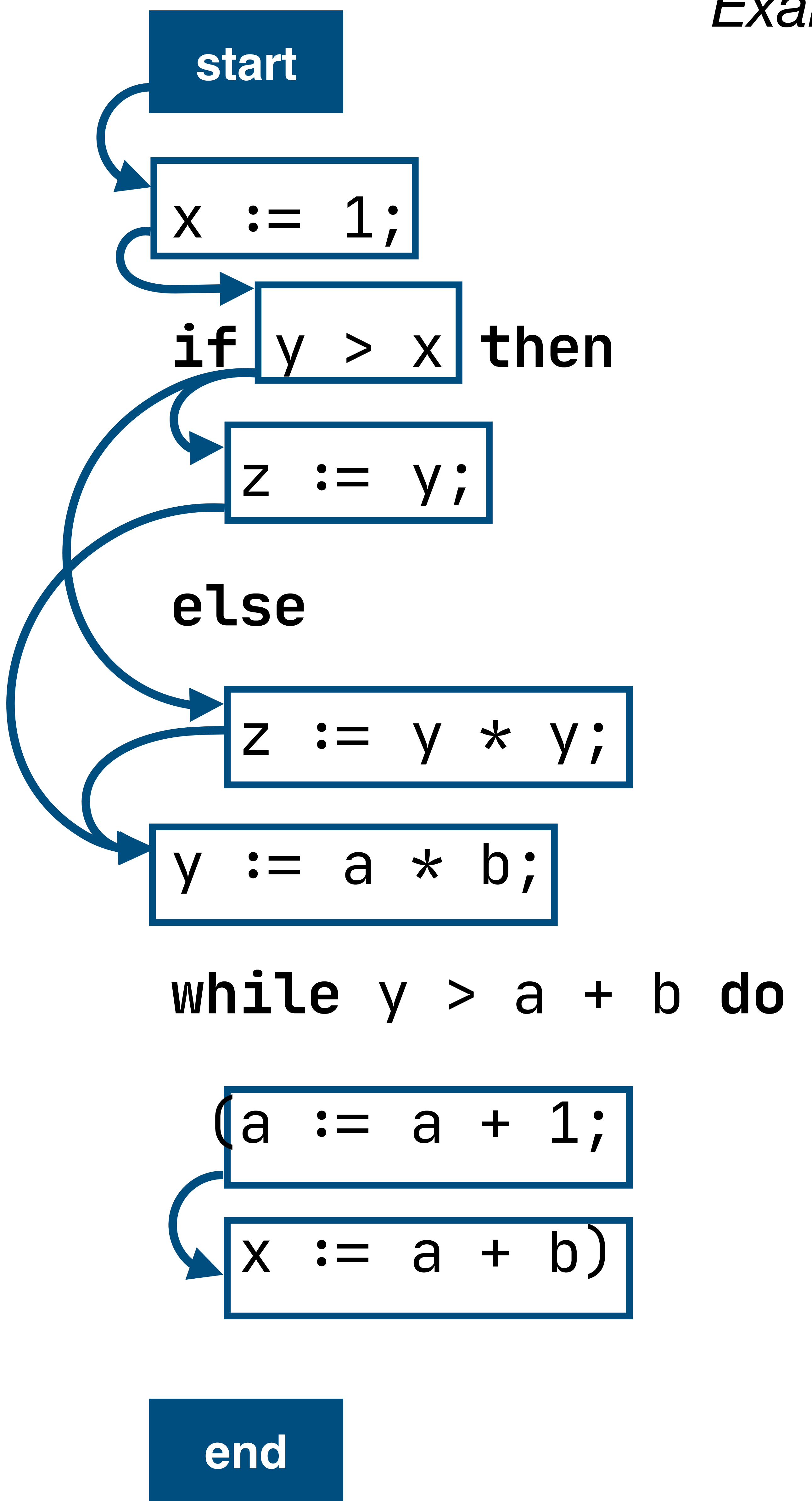
```

Example program



Control-Flow Graphs in FlowSpec

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Seq(s1, s2) =
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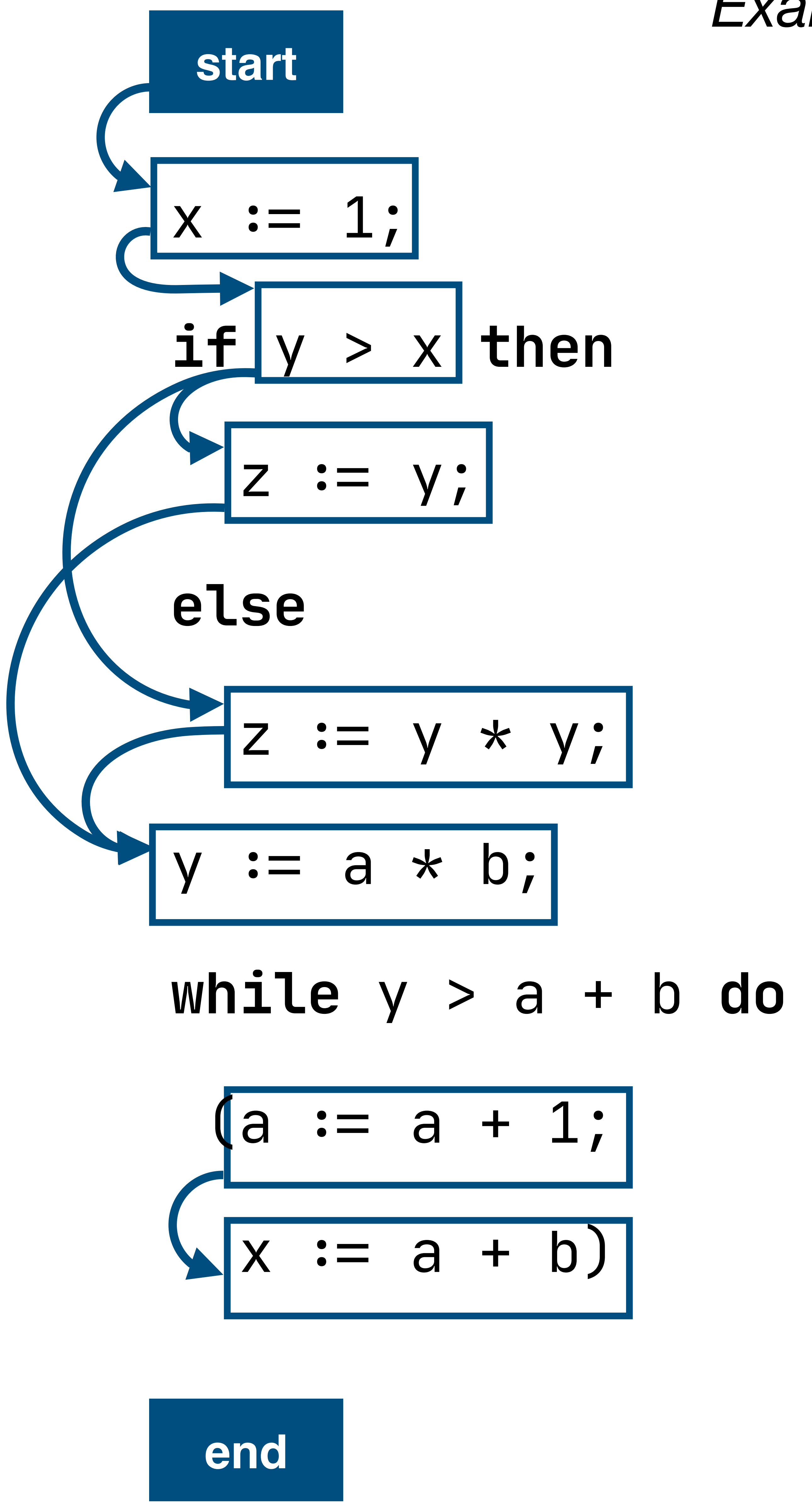
IfThenElse(c, t, e) =
  entry → node c → t → exit,
          node c → e → exit

While(c, b) =
  entry → node c → b → node c,
          node c → exit
```

Example program



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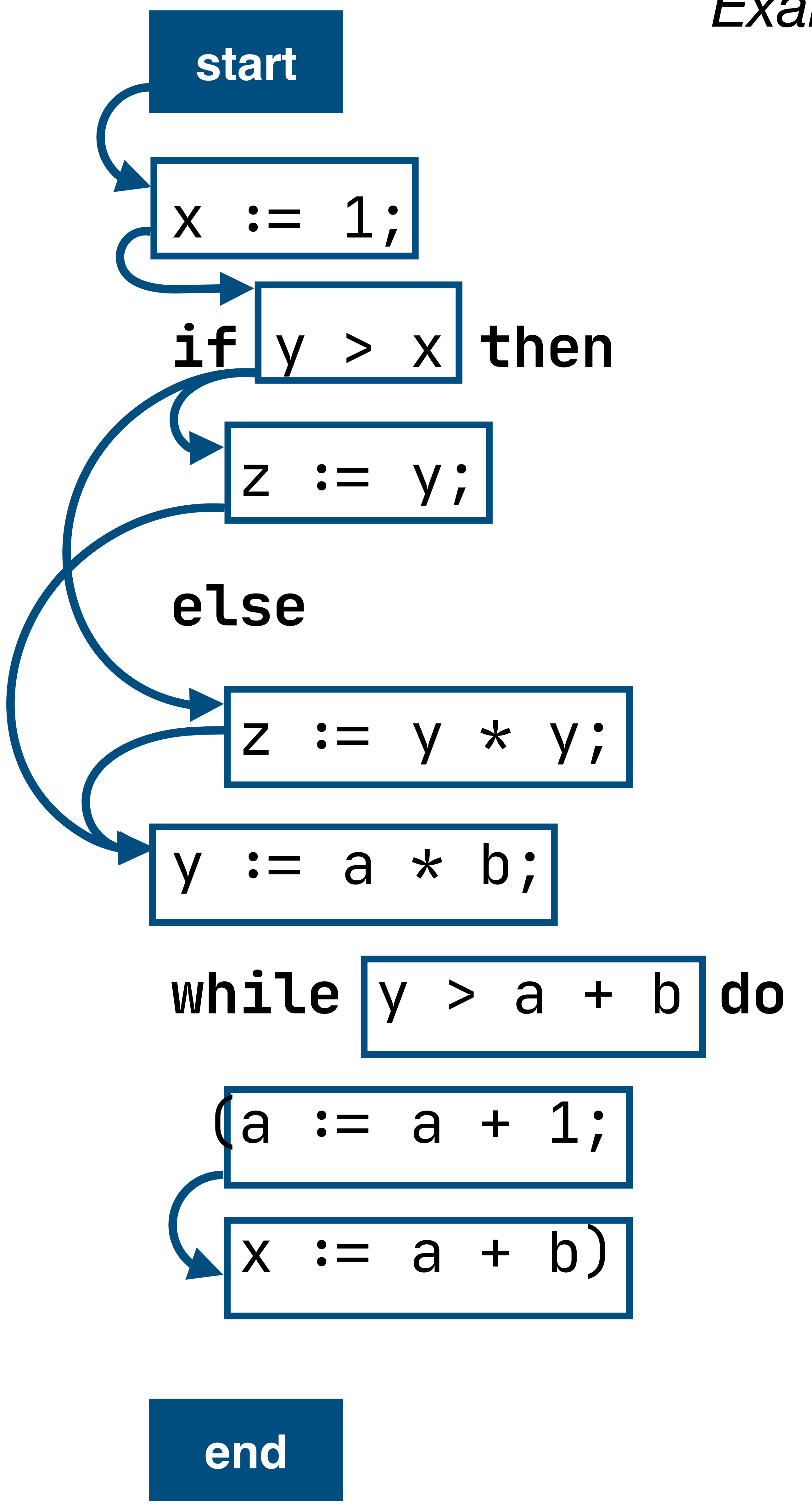
```

Example program



Control-Flow Graphs in FlowSpec

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While(c, b) =
  entry → node c → b → node c,
          node c → exit
```

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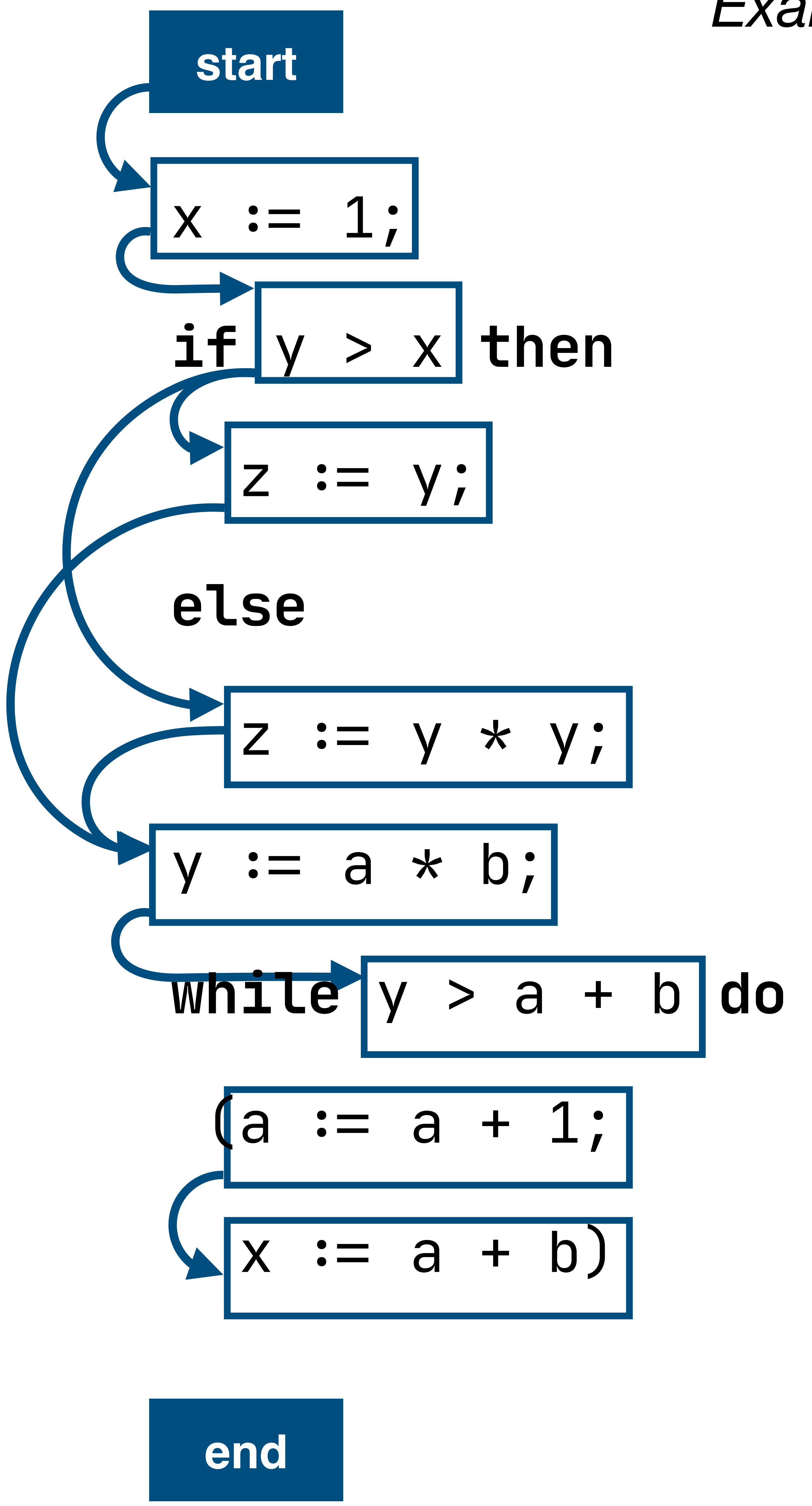

Control-Flow Graphs in FlowSpec

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root Mod(s) =
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node Assign(_, _)

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While(c, b) =
  entry → node c → b → node c,
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```

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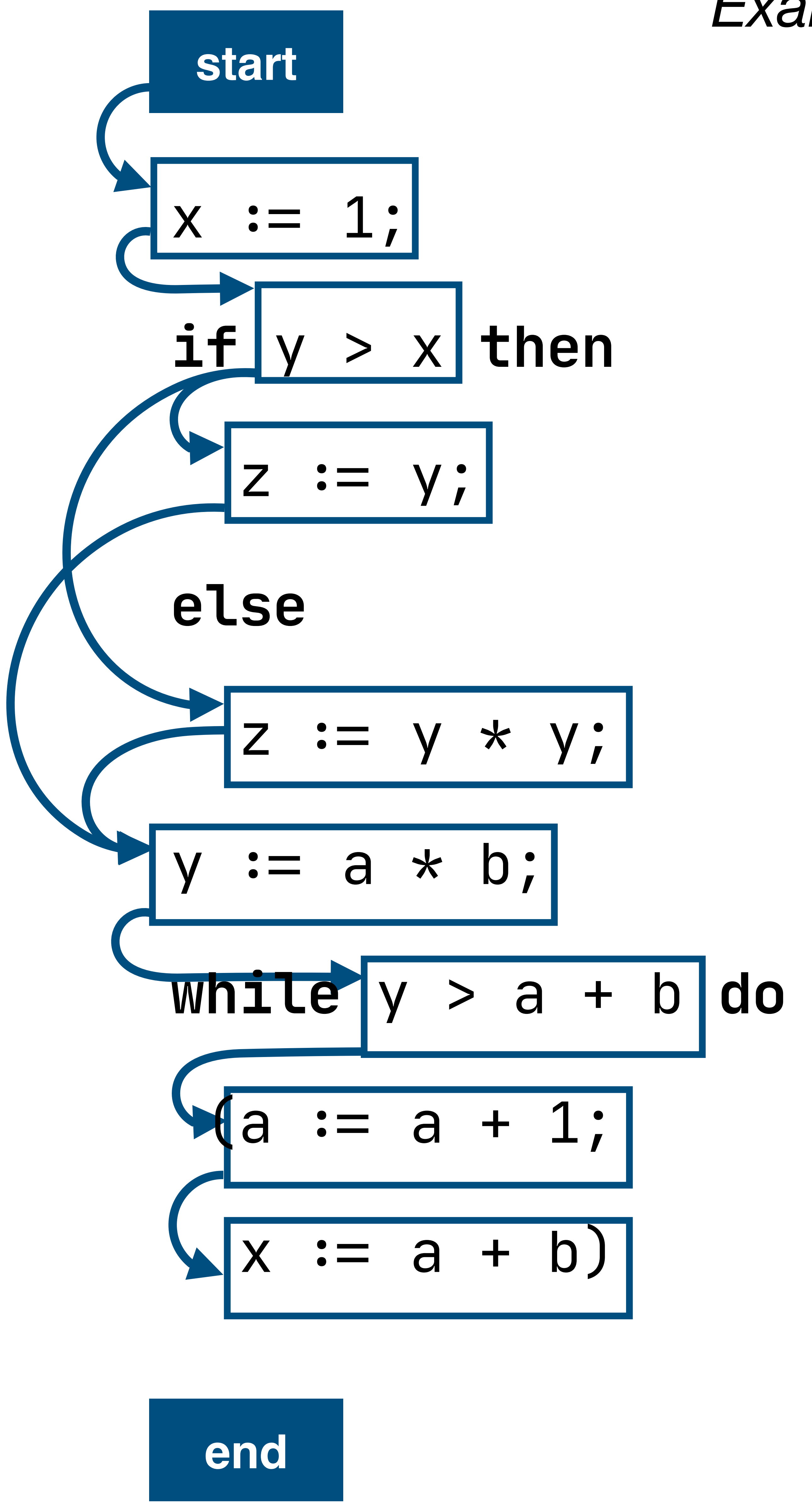

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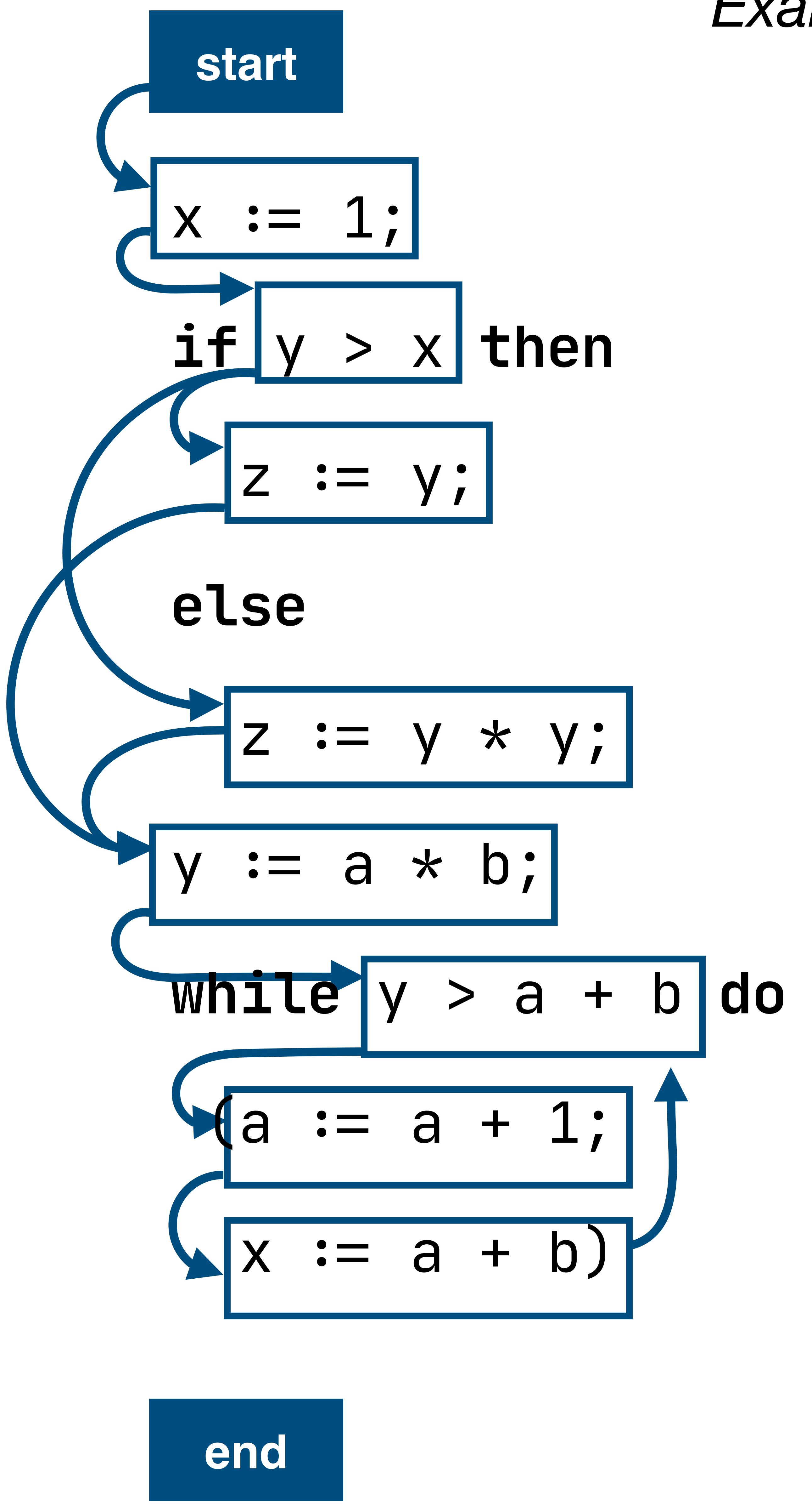

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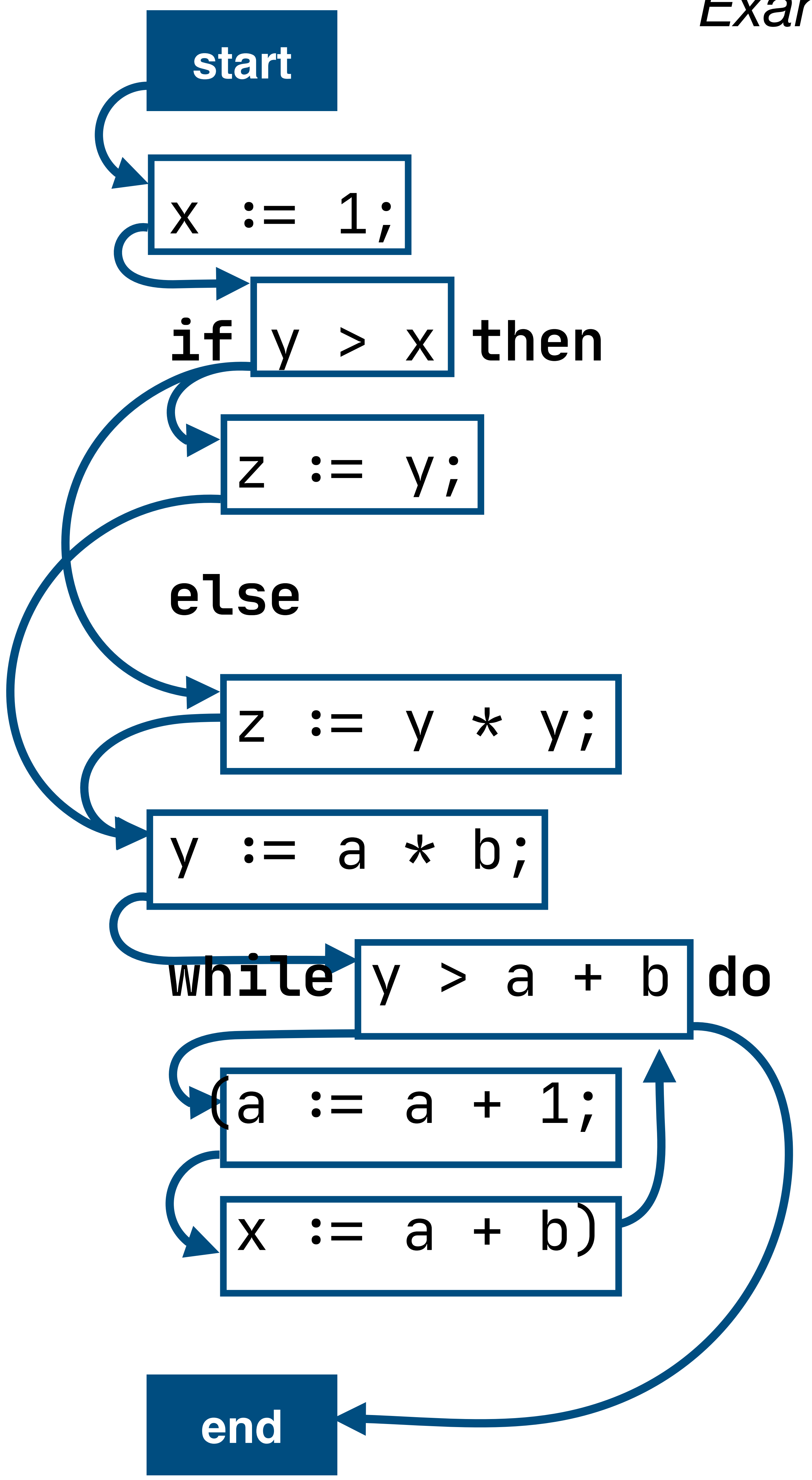
Example program



Data-Flow Rules

Data-Flow Rules

Define effect of control-flow graph nodes

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- Match an AST pattern on one side of a CFG edge

Data-Flow Rules

Define effect of control-flow graph nodes

- Match an AST pattern on one side of a CFG edge
- Propagate the information from the other side of the edge

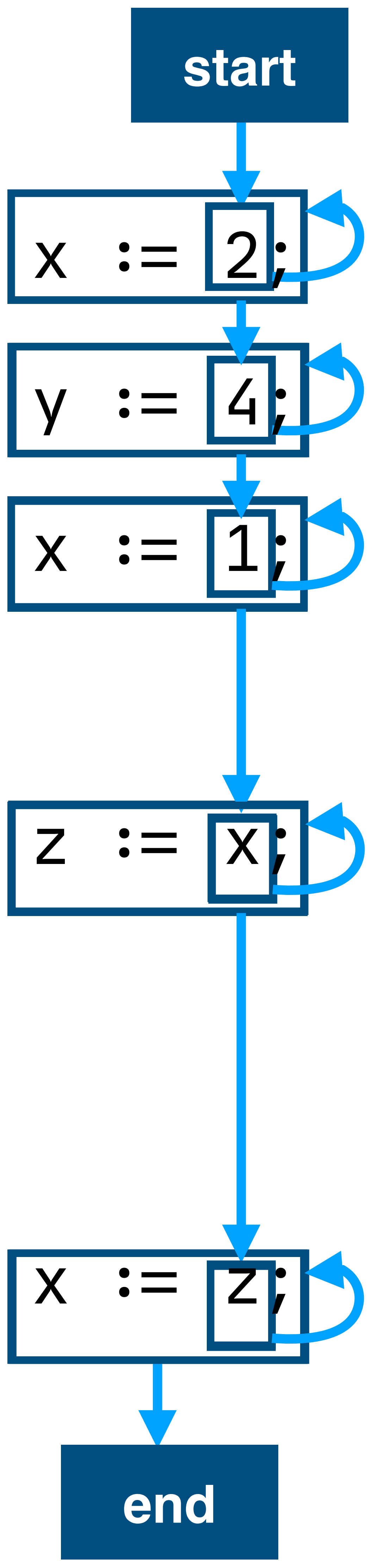
Data-Flow Rules

Define effect of control-flow graph nodes

- Match an AST pattern on one side of a CFG edge
- Propagate the information from the other side of the edge
- Adapt that information as the effect of the matched CFG node

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program



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properties
 live:     Set(name)
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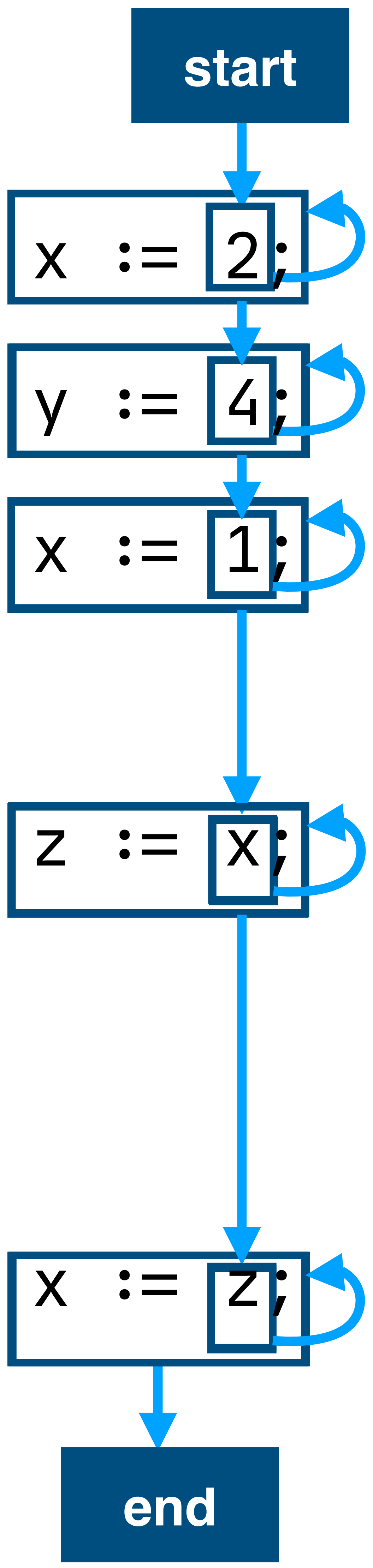

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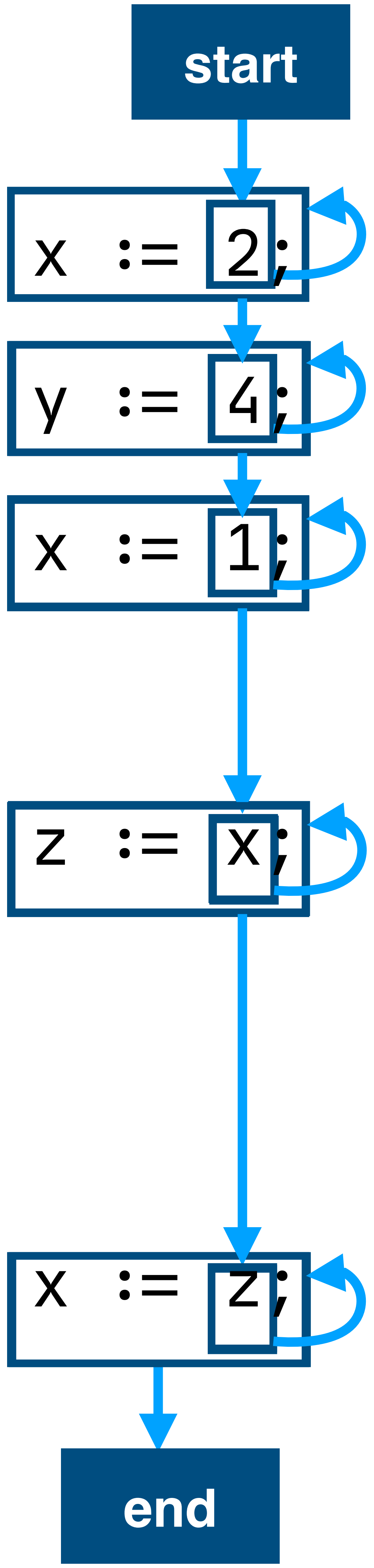

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
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 live(_.end) =
   {}
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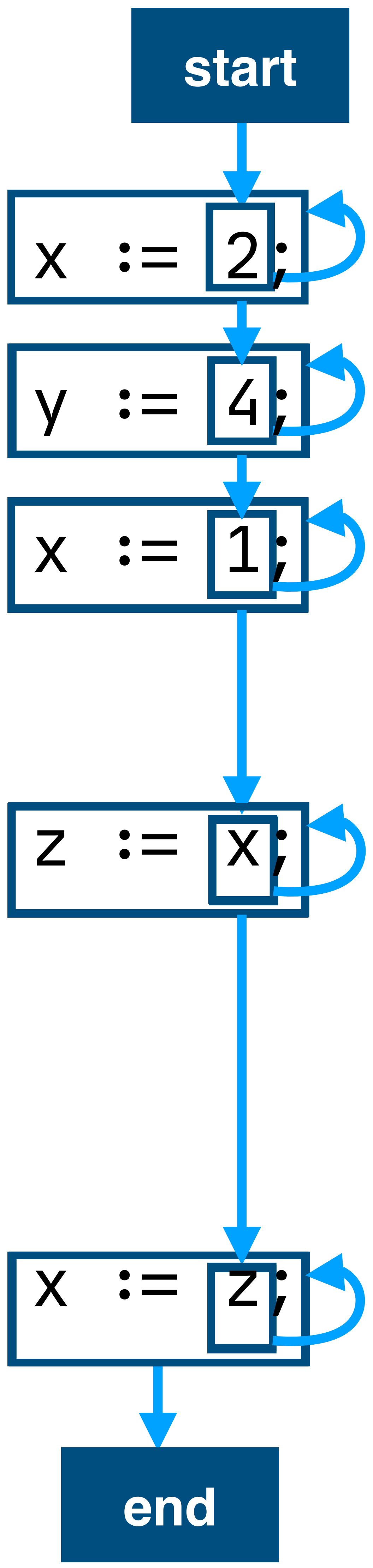

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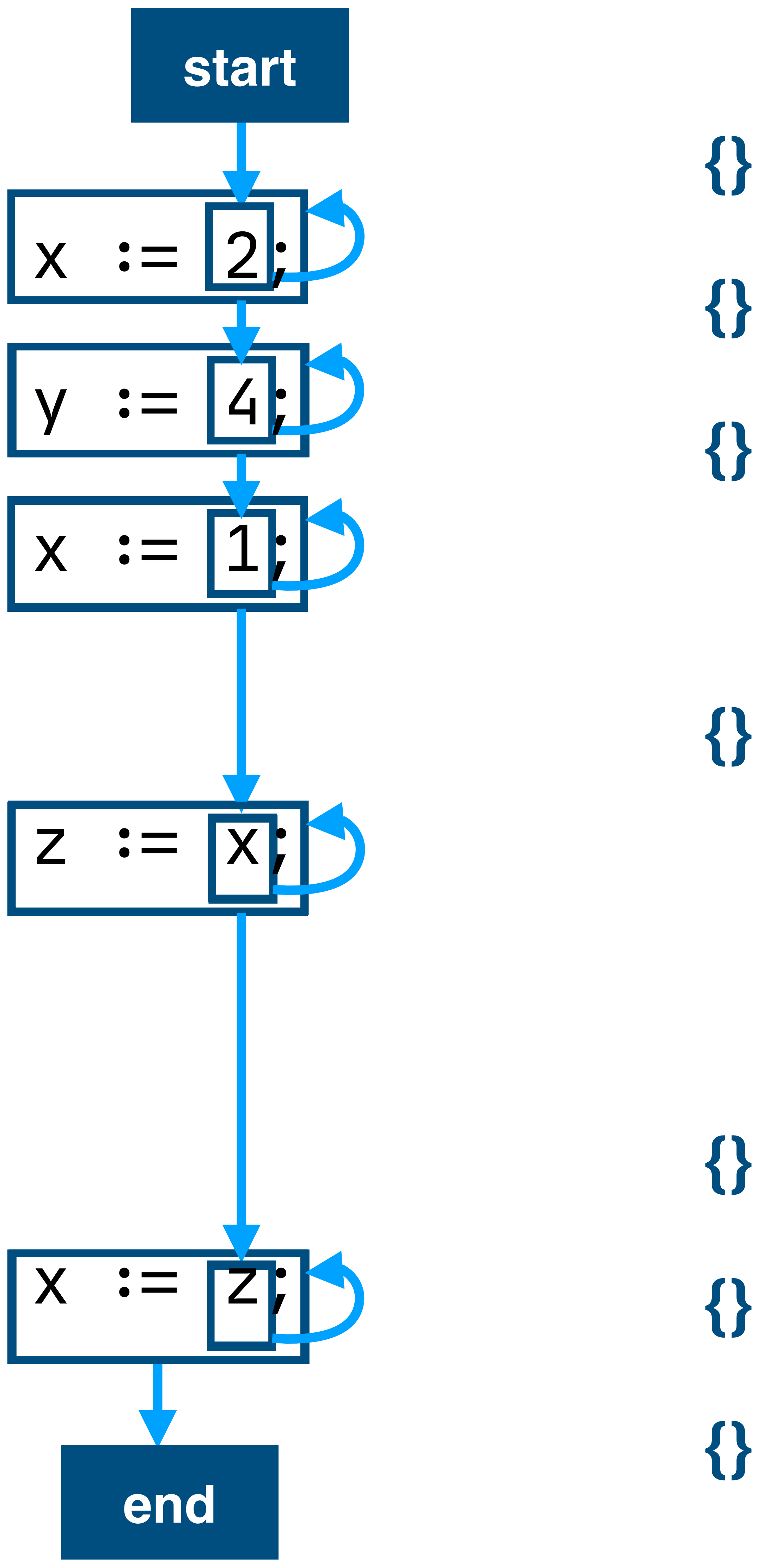

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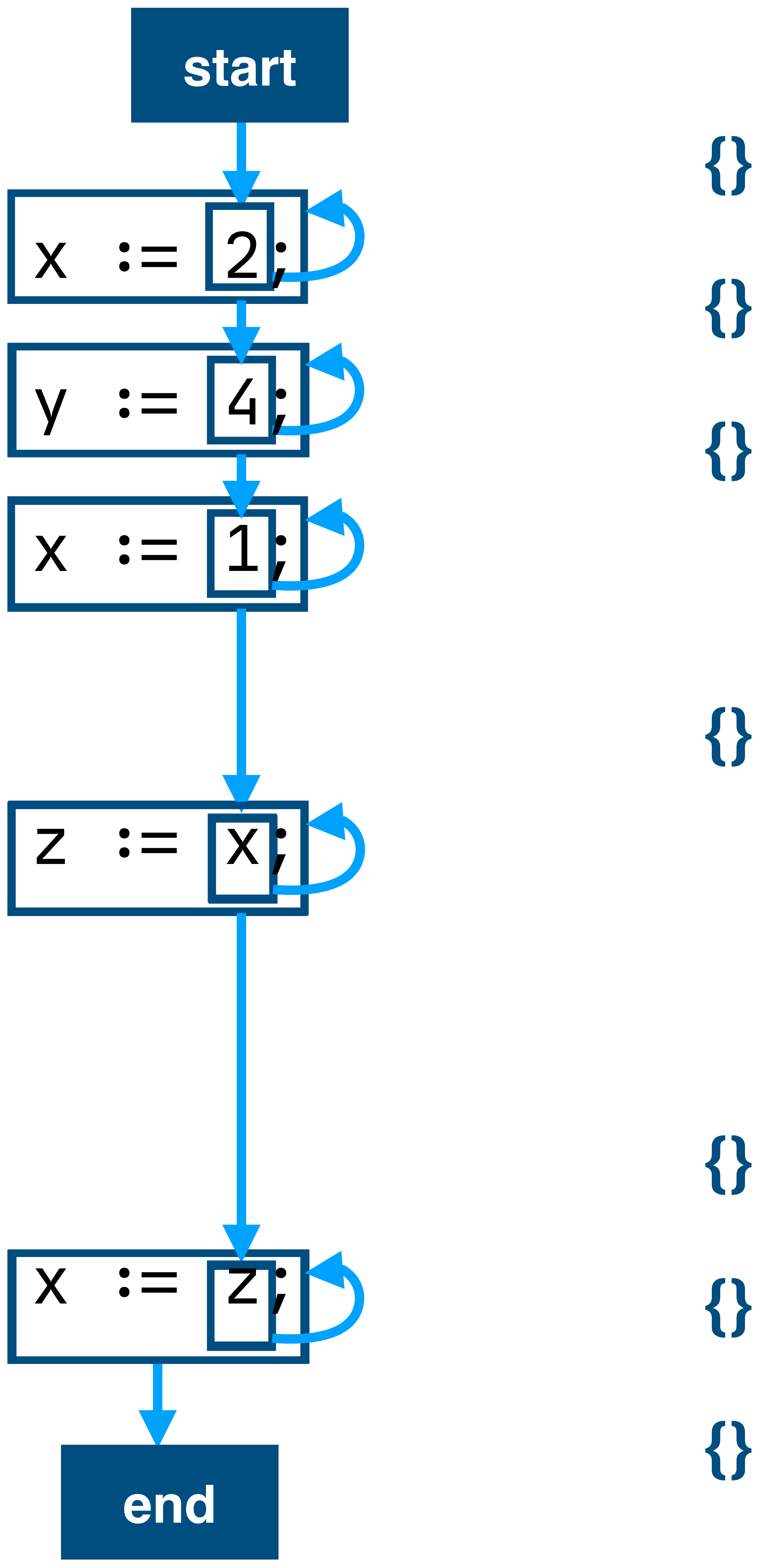

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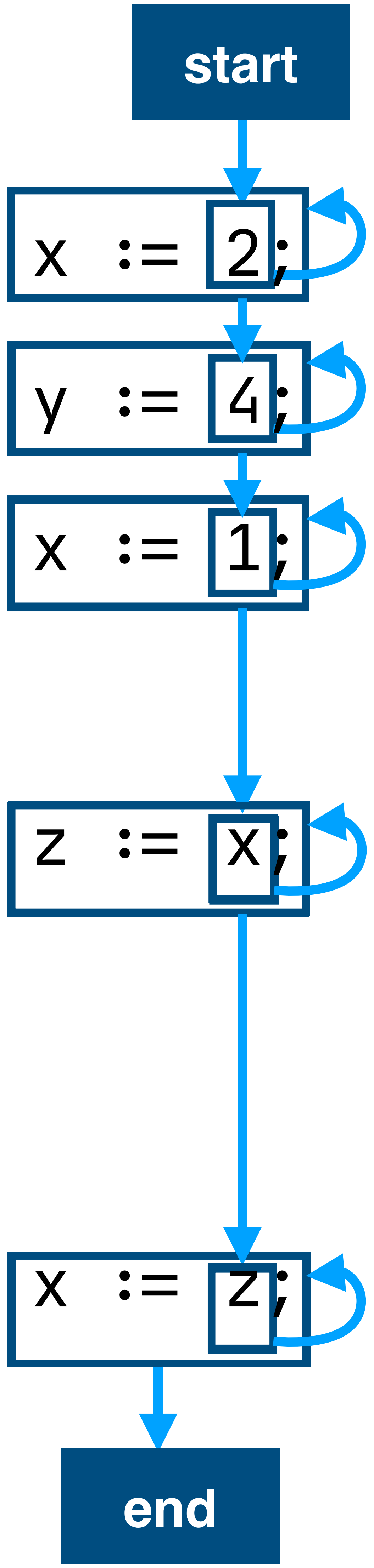

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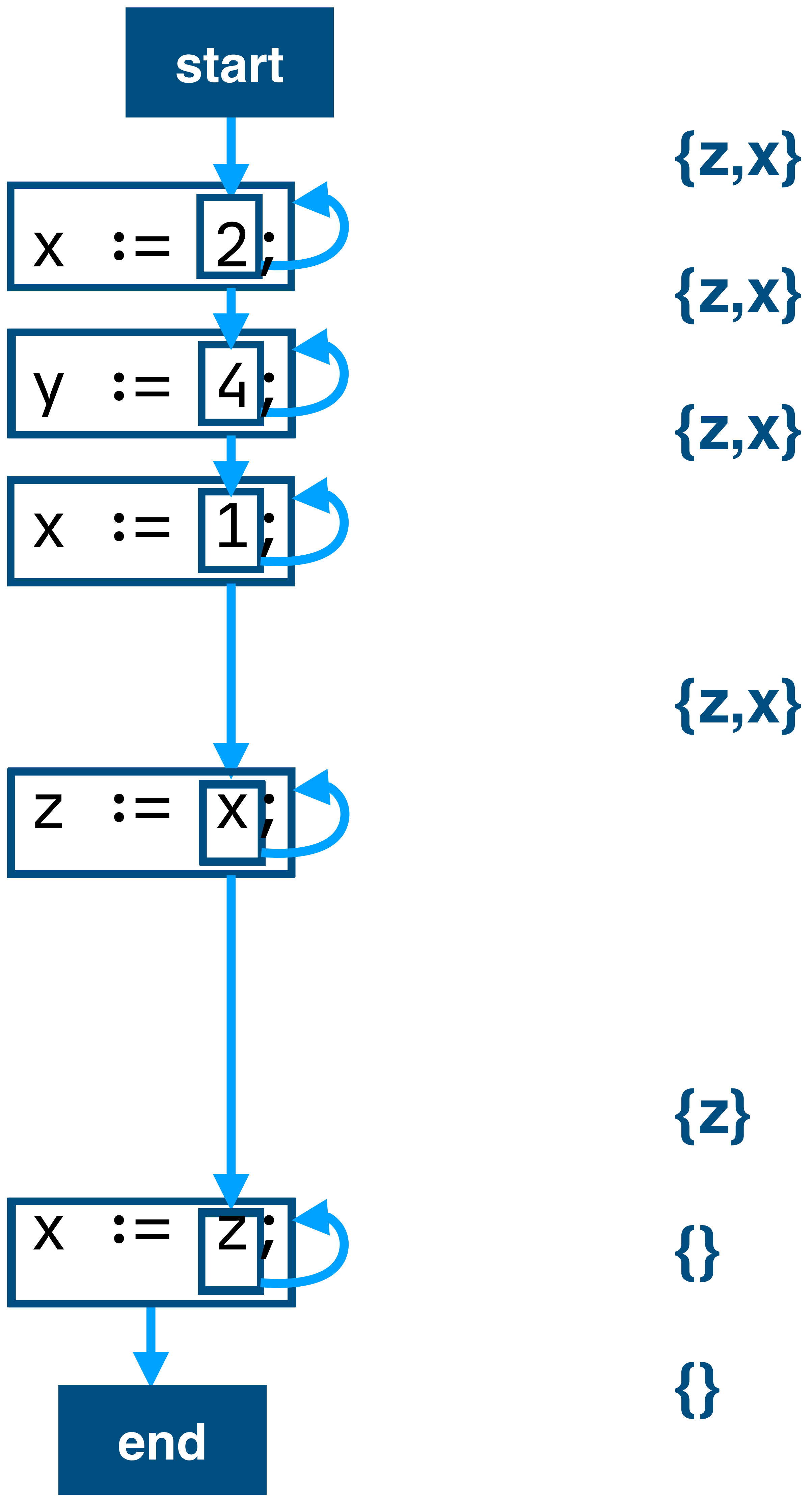

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   { m | m ← live(next), Var{n} ≠ m }

 live(_.end) =
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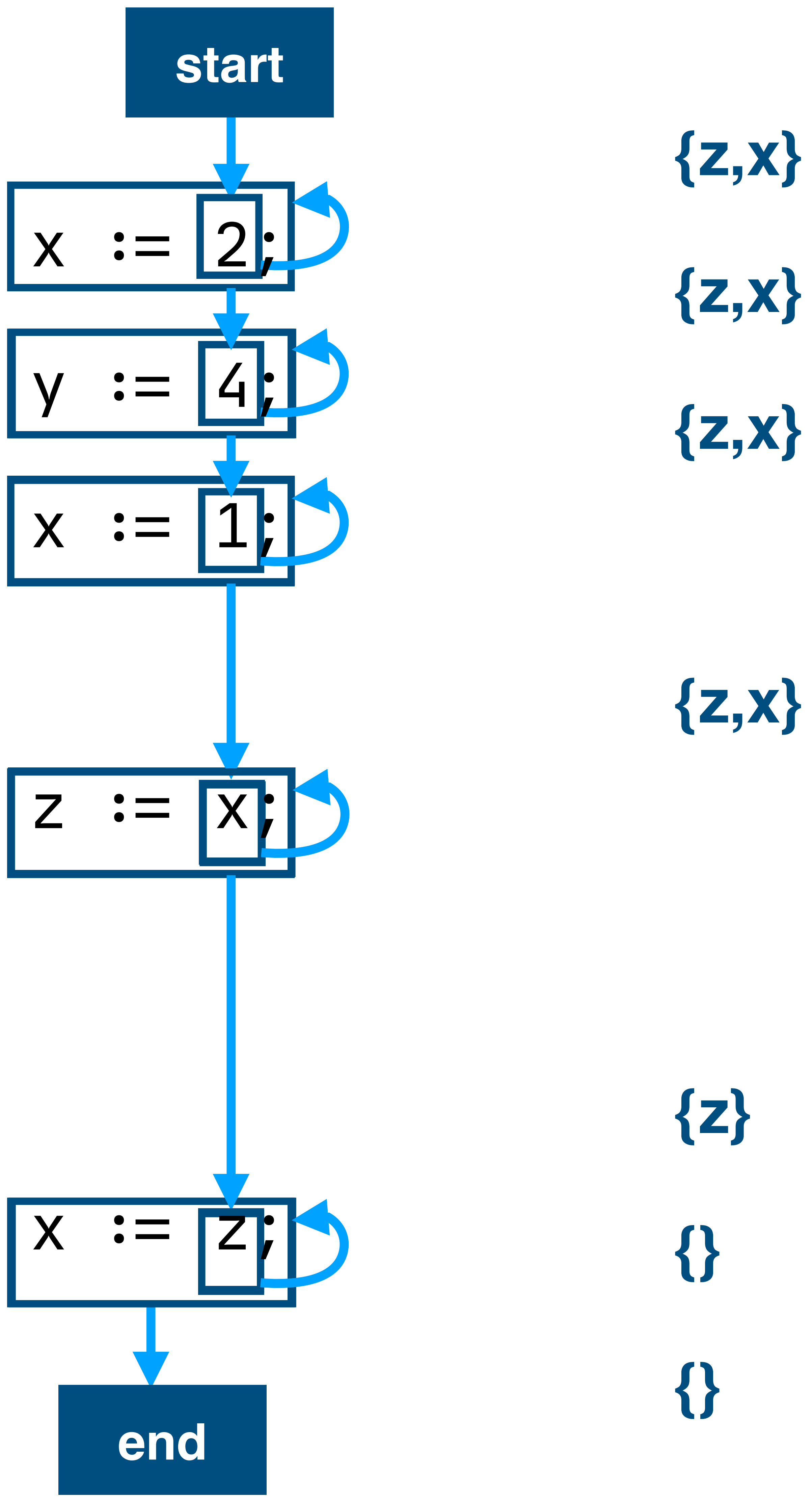

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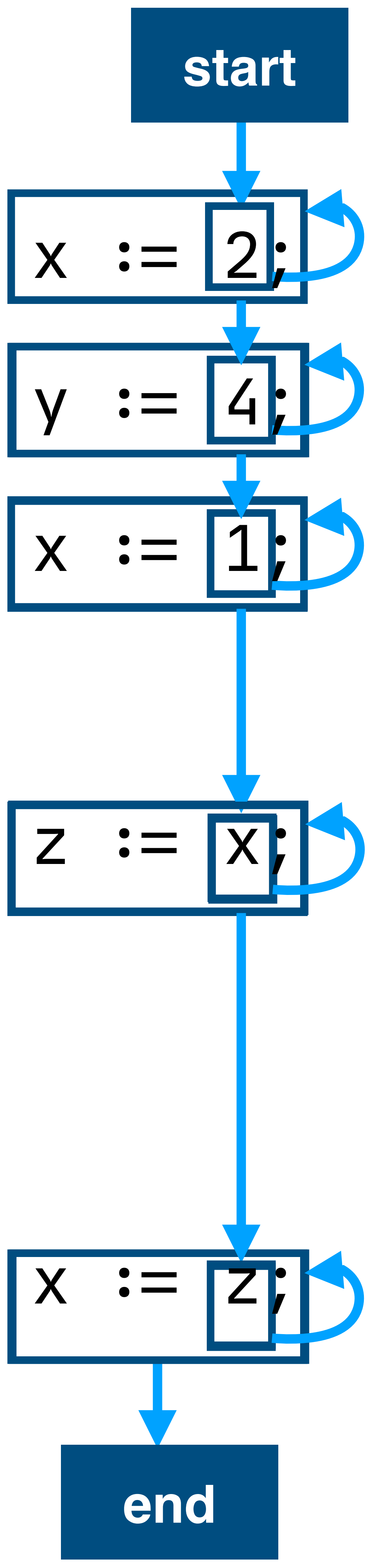

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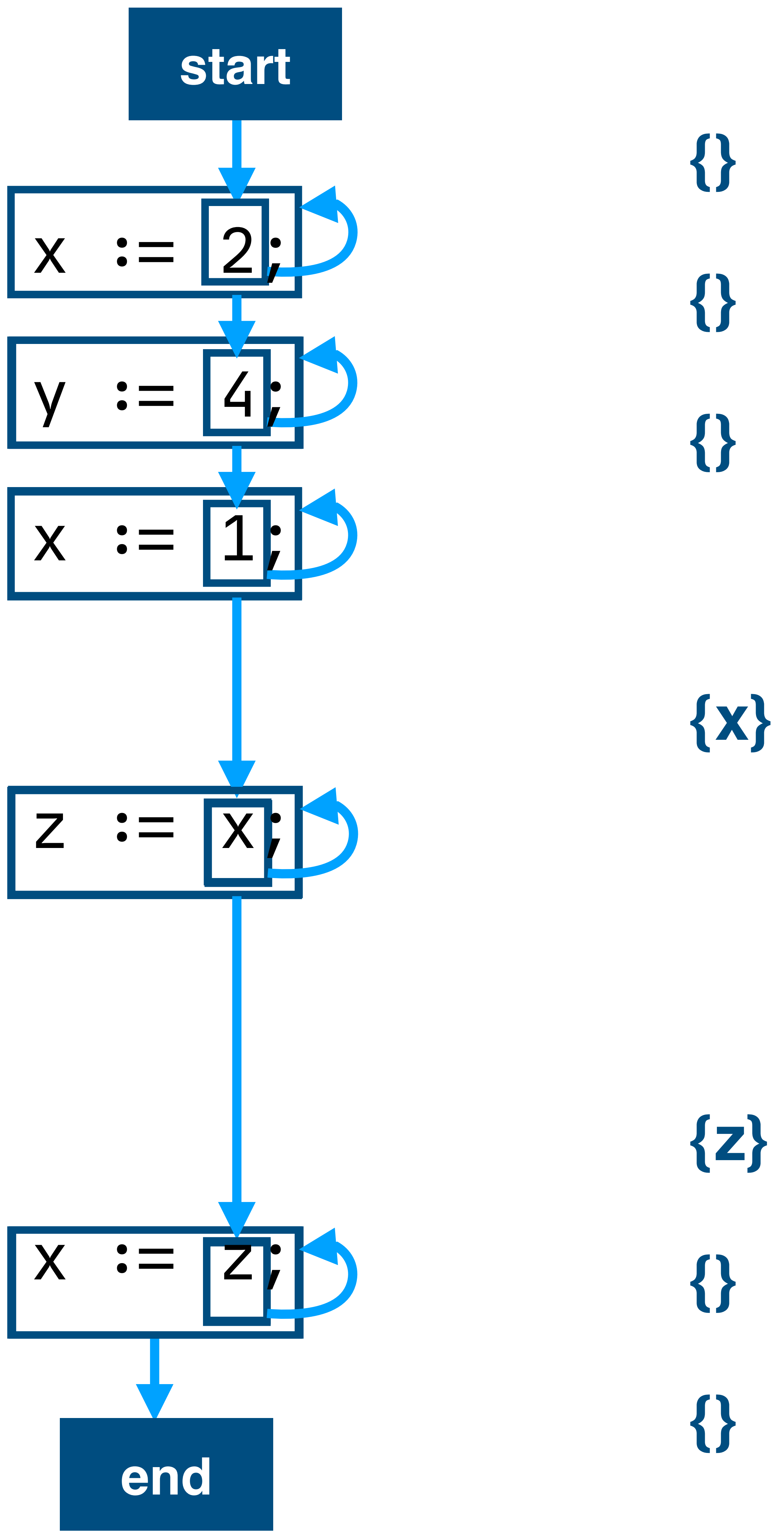

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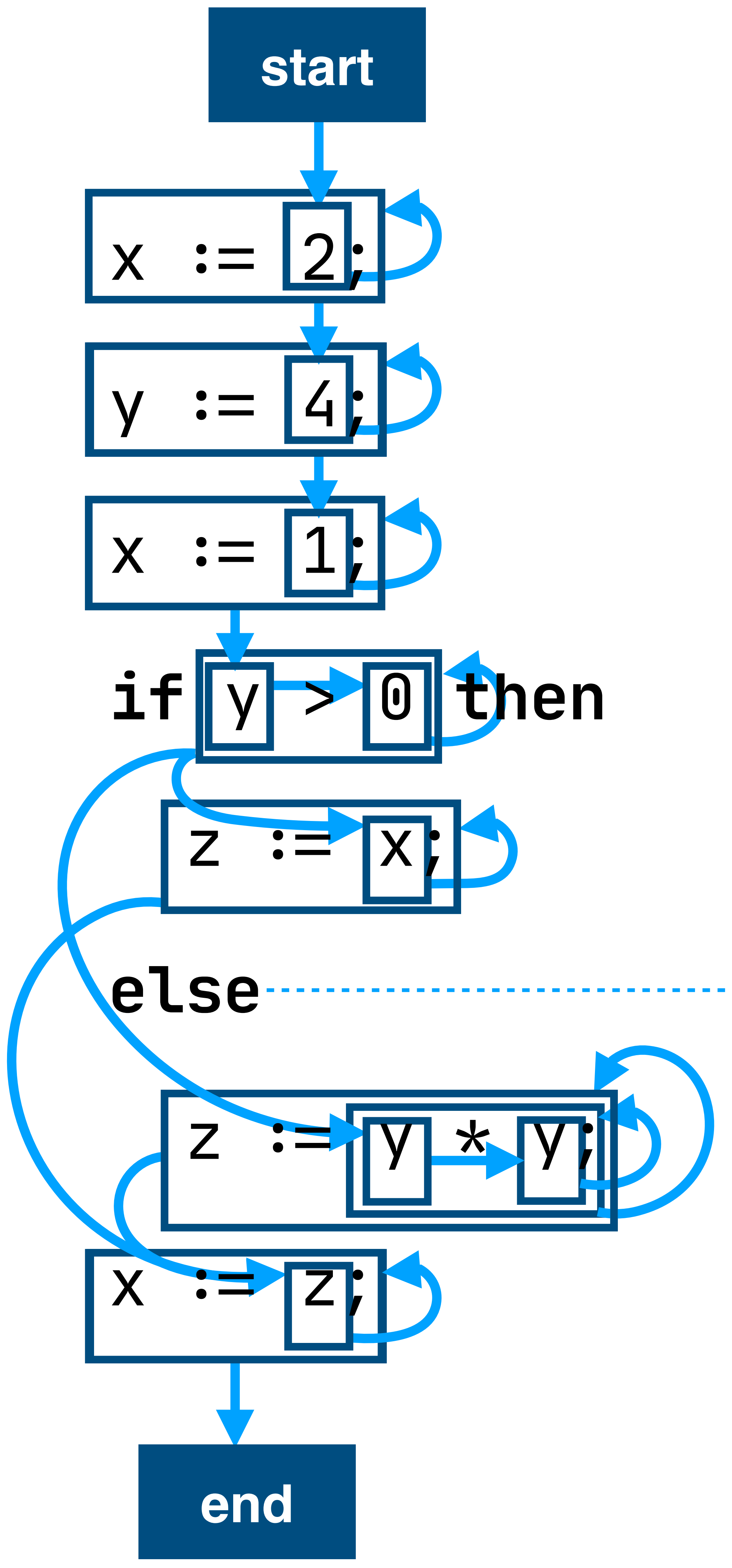

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 live(_.end) =
   {}
```

```
x := 2;

y := 4;

x := 1;

if y > 0 then

  z := x;

else

  z := y * y;

x := z;
```

Live Variables in FlowSpec

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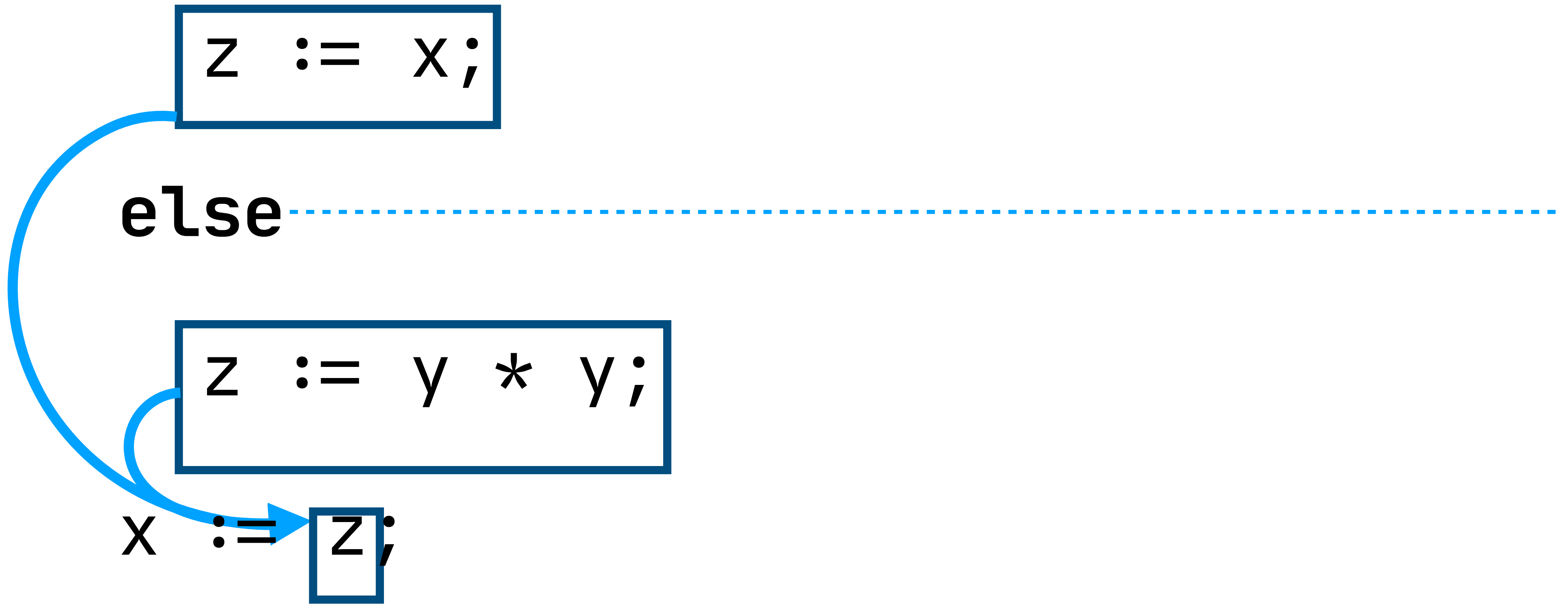

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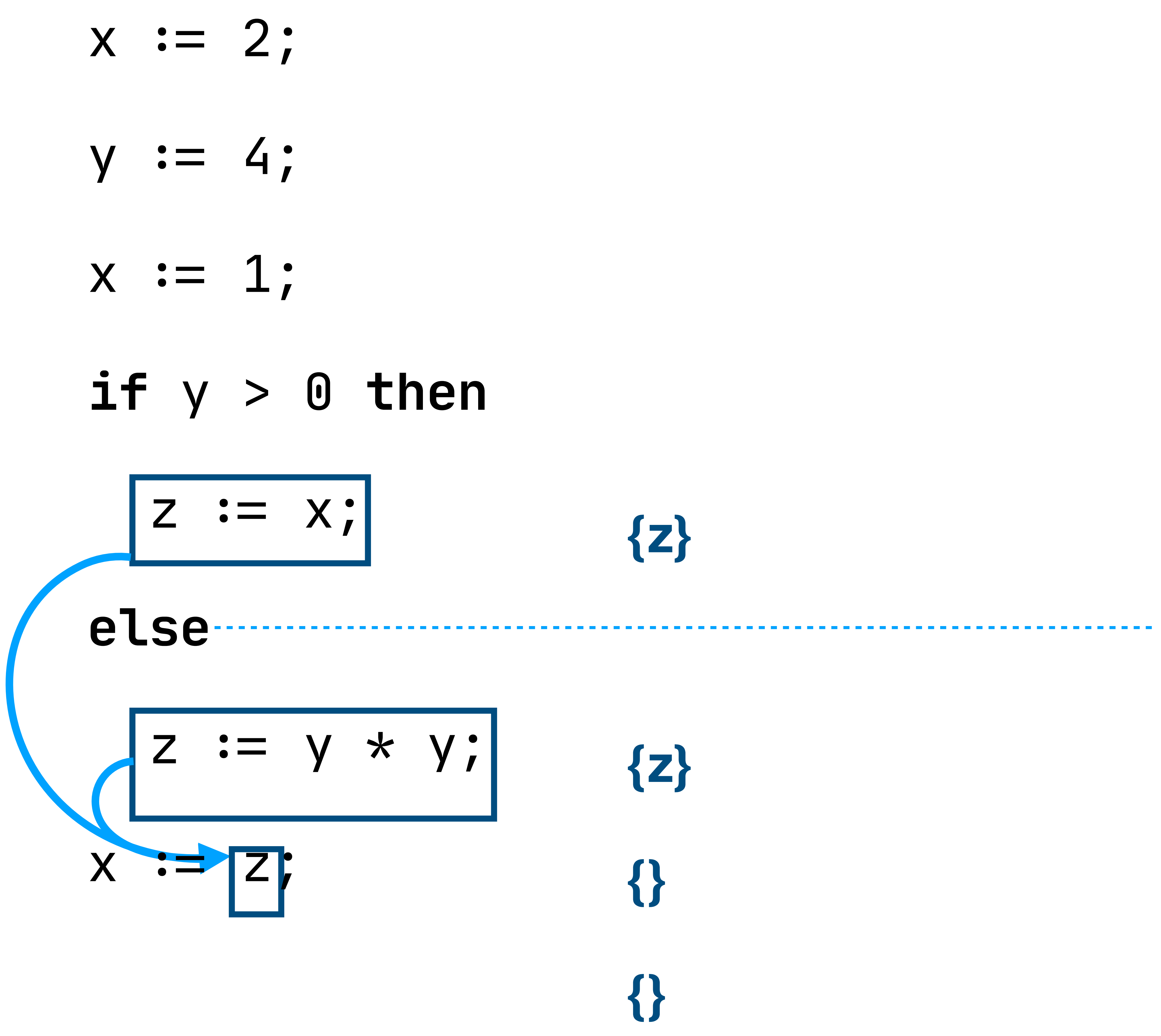

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x := 2;
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x := 1;
if y > 0 then
  z := x;         {z}
else
  z := y * y;     {z}
x := z;           {}
                  {}
```

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
 live:     Set(name)

property rules
 live(Ref(n) → next) =
   live(next) \/ {n}

 live(Assign(n, _) → next) =
   { m | m ← live(next), n ≠ m }

 live(_.end) =
   {}
```

```
x := 2;

y := 4;

x := 1;

if y > 0 then

  z := x;                  {z}

else

  z := y * y;              {z}

x := z;                    {}

                           {}
```

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
 live:     Set(name)

property rules
 live(Ref(n) → next) =
   live(next) \/ {n}

 live(Assign(n, _) → next) =
   { m | m ← live(next), n ≠ m }

 live(_.end) =
   {}
```

```
x := 2;

y := 4;

x := 1;

if y > 0 then        {x}

  z := x;            {z}

else                 {y}

  z := y * y;        {z}

x := z;              {}

                     {}
```

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
 live: MaySet(name)

property rules
 live(Ref(n) → next) =
   live(next) \/ {n}

 live(Assign(n, _) → next) =
   { m | m ← live(next), n ≠ m }

 live(_.end) =
   {}
```

```
x := 2;

y := 4;

x := 1;

if y > 0 then
                    {x}
  z := x;
                    {z}
else
                    {y}
  z := y * y;
                    {z}
x := z;
                    {}

                    {}
```

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
 live: MaySet(name)

property rules
 live(Ref(n) → next) =
   live(next) \/ {n}

 live(Assign(n, _) → next) =
   { m | m ← live(next), n ≠ m }

 live(_.end) =
   {}
```

```
x := 2;

y := 4;

x := 1;

if y > 0 then

  z := x;

else

  z := y * y;

x := z;
```

{x,y}

{x}

{z}

{y}

{z}

{}

{}

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
x := a + b
y := a * b
while y > a + b do (
    a := a + 1;
    x := a + b
)
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)
```

```
x := a + b

y := a * b

while y > a + b do (

    a := a + 1;

    x := a + b

)
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

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```

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```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

property rules



 available(_.start) =
   {}
```

```
x := a + b

y := a * b

while y > a + b do (

    a := a + 1;

    x := a + b

)
```

Available Expressions in FlowSpec

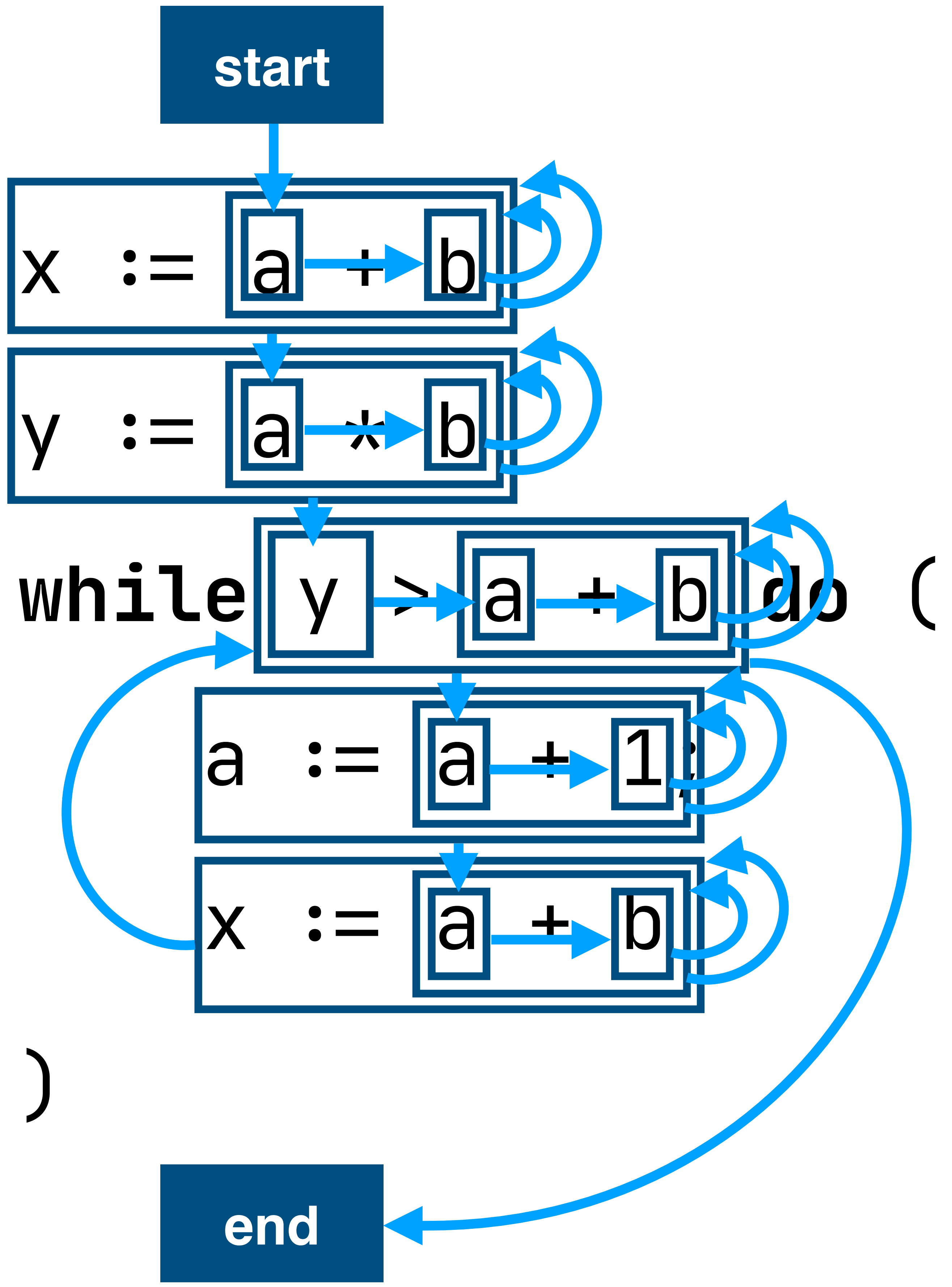
An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

property rules



 available(_.start) =
   {}
```



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properties
 available: MustSet(term)

property rules




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   {}
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```
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    x := a + b

)
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

property rules




 available(_.start) =
   {}
```

```
                                {}
x := a + b                      {}
y := a * b                      {}
while y > a + b do (            {}
    a := a + 1;                 {}
    x := a + b                  {}
)                               {}
                                {}
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

property rules



 available(_.start) =
   {}
```

```
x := a + b
y := a * b
while y > a + b do (
    a := a + 1;
    x := a + b
)
```

{}
{}
{}
{}
{}
{}
{}
{}
{}

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

property rules

 available(prev -> Assign(n, e)) =
   { expr |
     expr <- available(prev) \/ {e},
     !(n in reads(expr)) }

 available(_.start) =
   {}
```

```
                                  {}
x := a + b                        {}
y := a * b                        {}
while y > a + b do (              {}
    a := a + 1;                   {}
    x := a + b                    {}
)                                 {}
                                  {}
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

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properties
 available: MustSet(term)

property rules

 available(prev → Assign(n, e)) =
   { expr |
     expr ← available(prev) \/ {e},
     !(n in reads(expr)) }

 available(_.start) =
   {}
```

```
                                   {}
x := a + b                         {a+b}
y := a * b                         {a+b,a*b}
while y > a + b do (               {a+b,a*b}
    a := a + 1;                    {}
    x := a + b                     {a+b}
)
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
properties
 available: MustSet(term)

property rules

 available(prev → Assign(n, e)) =
   { expr |
     expr ← available(prev) \/ {e},
     !(n in reads(expr)) }

 available(_.start) =
   {}
```

```
                              {}
x := a + b                    {a+b}
y := a * b                    {a+b,a*b}
while y > a + b do (          {a+b}
    a := a + 1;               {}
    x := a + b                {a+b}
)                             {a+b}
```

Summary

Summary: Data-Flow Analysis Specification

Summary: Data-Flow Analysis Specification

Control-Flow

Summary: Data-Flow Analysis Specification

Control-Flow

- Order of execution

Summary: Data-Flow Analysis Specification

Control-Flow

- Order of execution
- Reasoning about what is reachable

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- Flow of data through a program

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- Flow of data through a program
- Reasoning about data, and dependencies between data

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- Control-Flow rules to construct the graph

Summary: Data-Flow Analysis Specification

Control-Flow

- Order of execution
- Reasoning about what is reachable

Data-Flow

- Flow of data through a program
- Reasoning about data, and dependencies between data

FlowSpec

- Control-Flow rules to construct the graph
- Annotate with information from analysis by Data-Flow rules

Next

From Specification to Implementation

Traditional Kill/Gen Sets

Available Expressions

"An **expression** is **available** if it *must* have already been computed, and not later modified, on all paths to the program point"

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

Available Expressions

"An **expression** is **available** if it *must* have already been computed, and not later modified, on all paths to the program point"

kill(Assign(var, e1)) :=
{ e2 ∈ **AllAE** | var ∈ **FV**(e2) }

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

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previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

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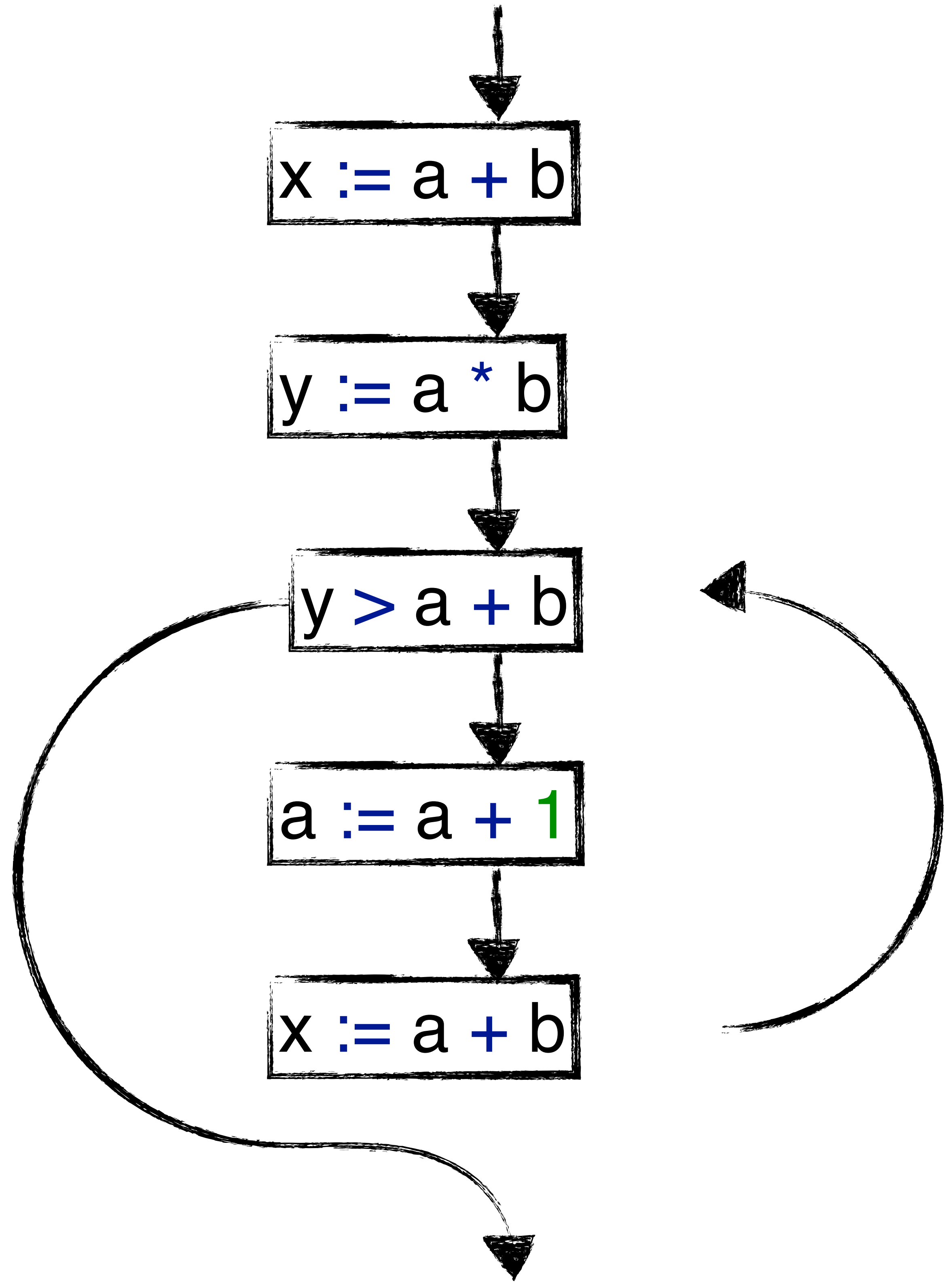

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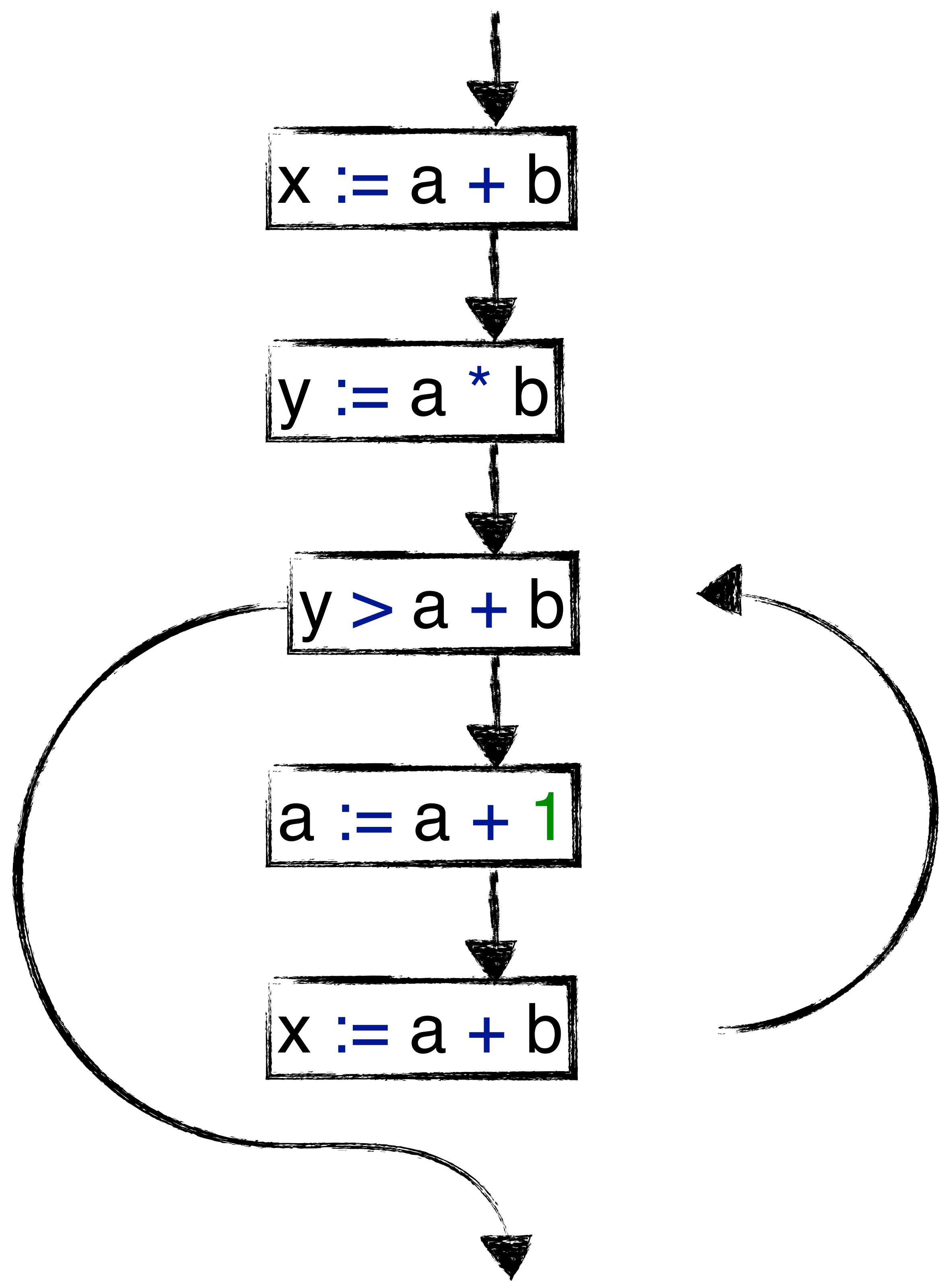

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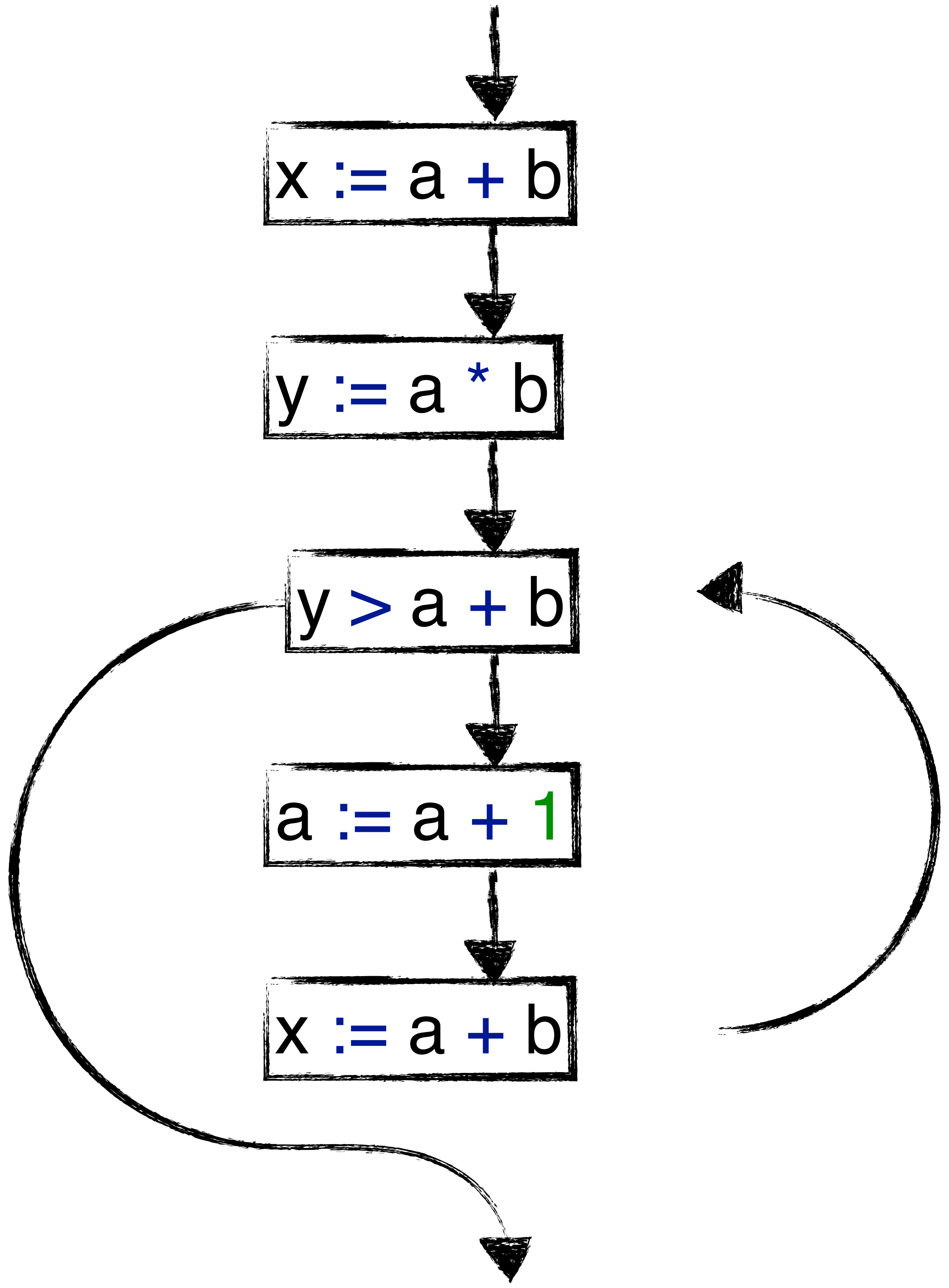

{}

{a + b}

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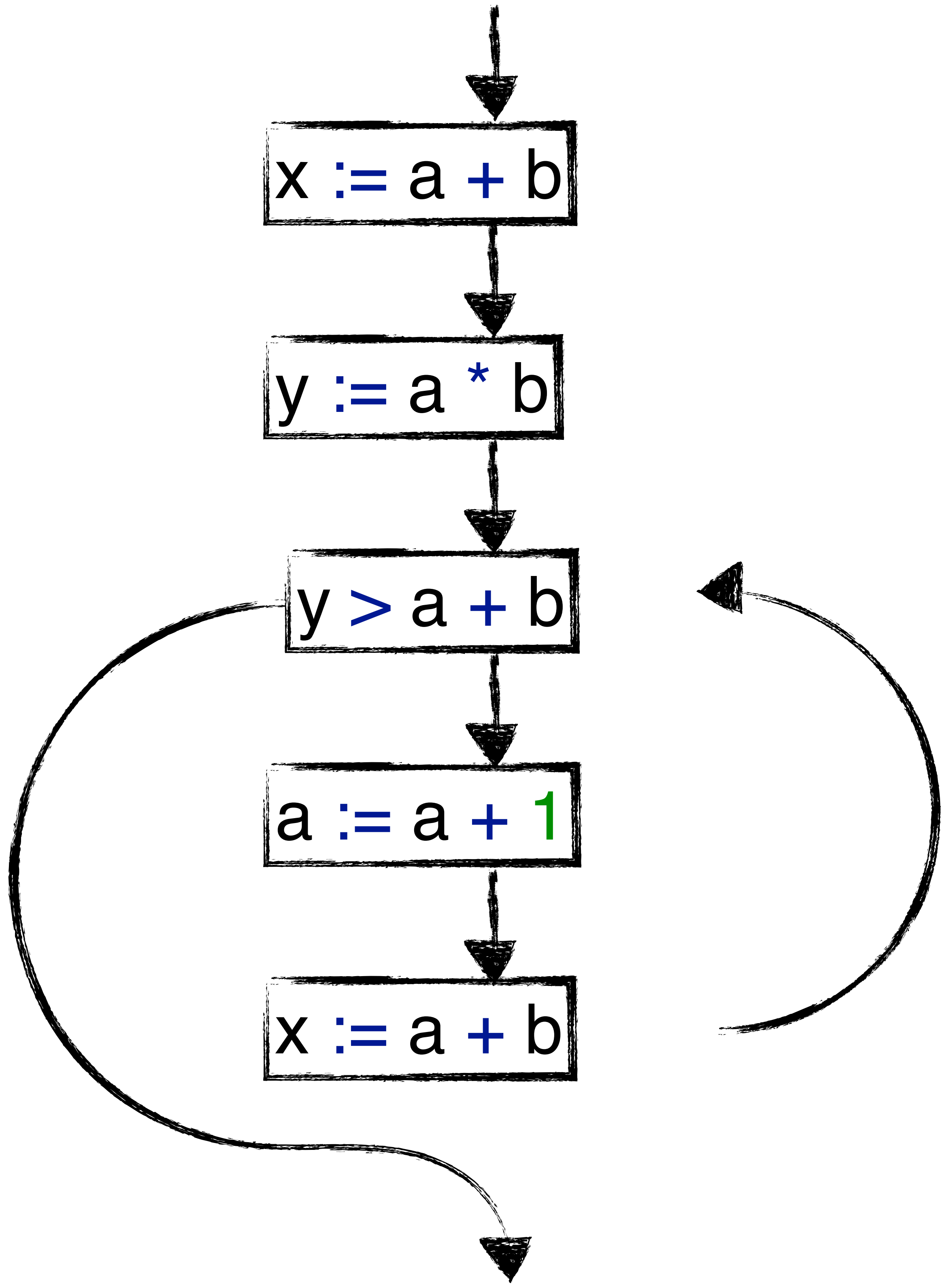

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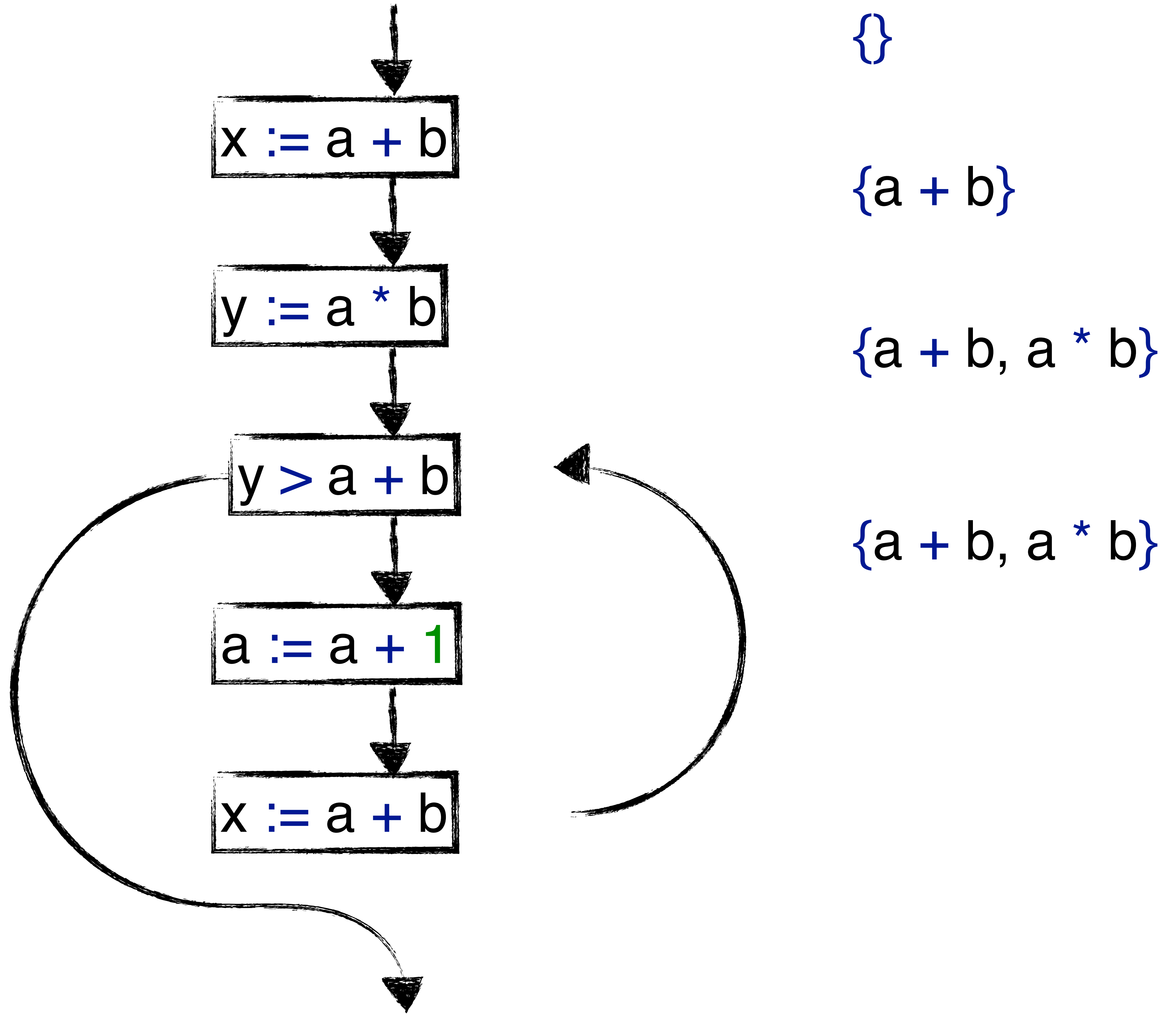

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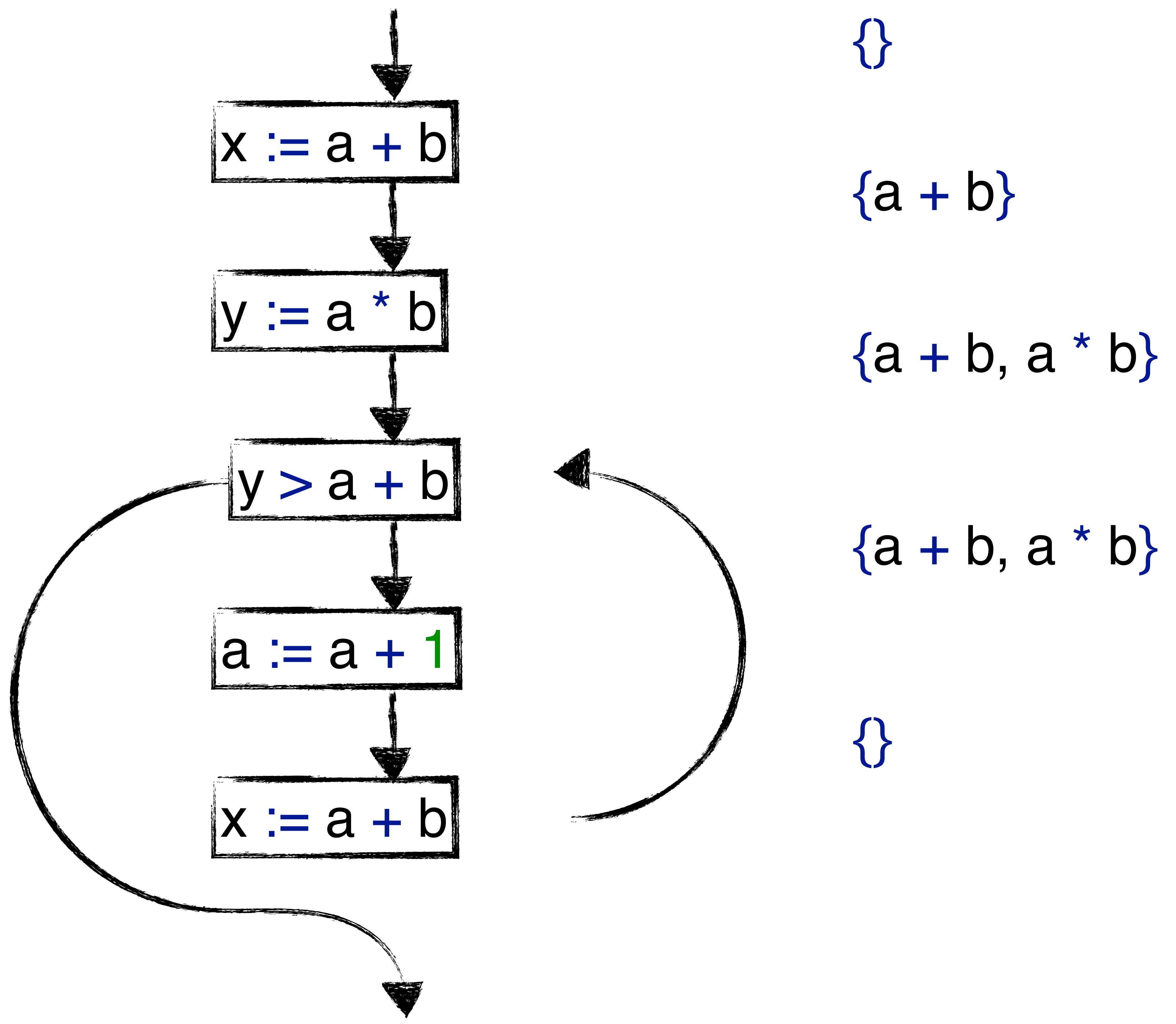

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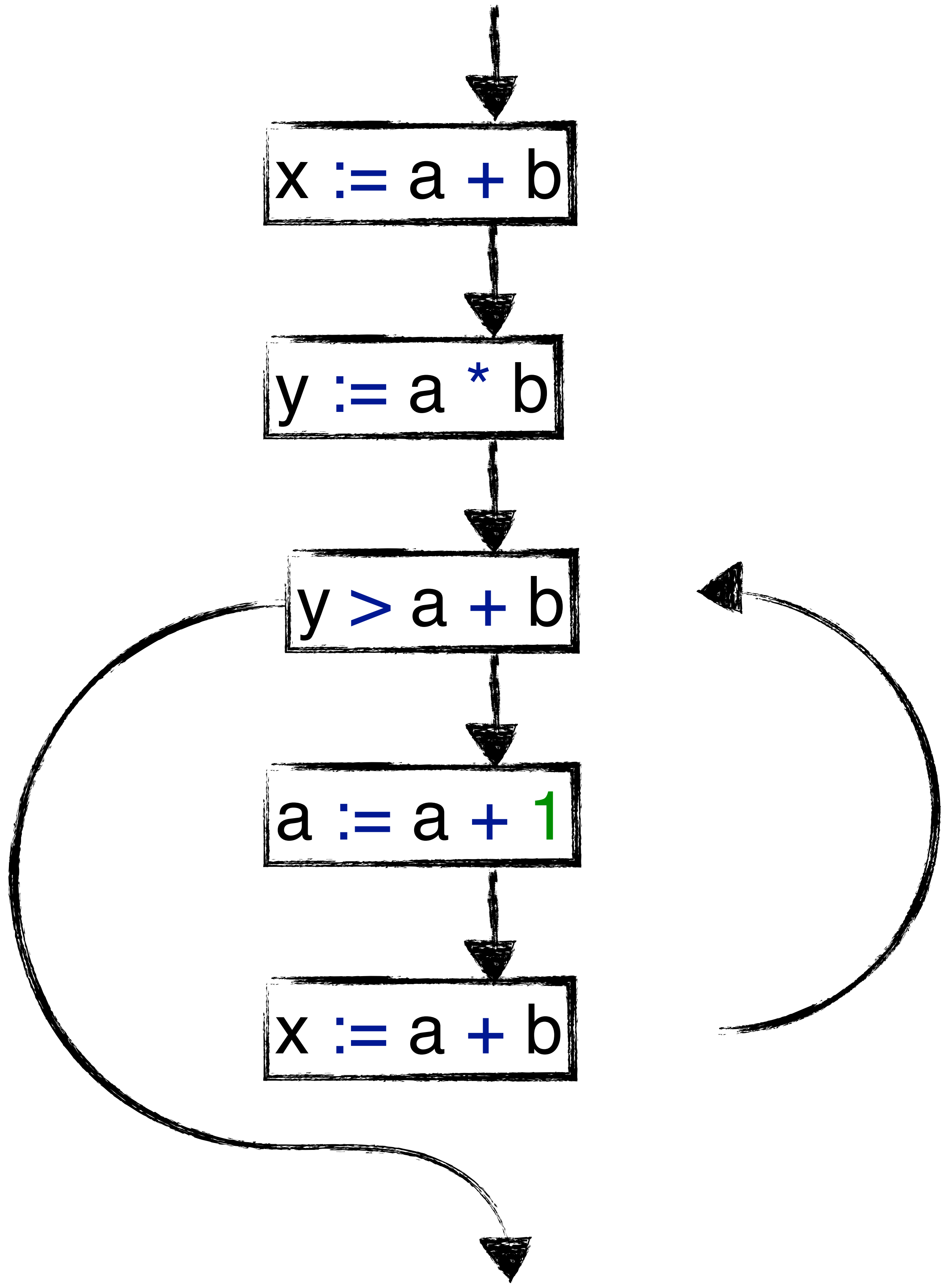

{}

{a + b}

{a + b, a * b}

{a + b, a * b}

{}

{a + b}

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

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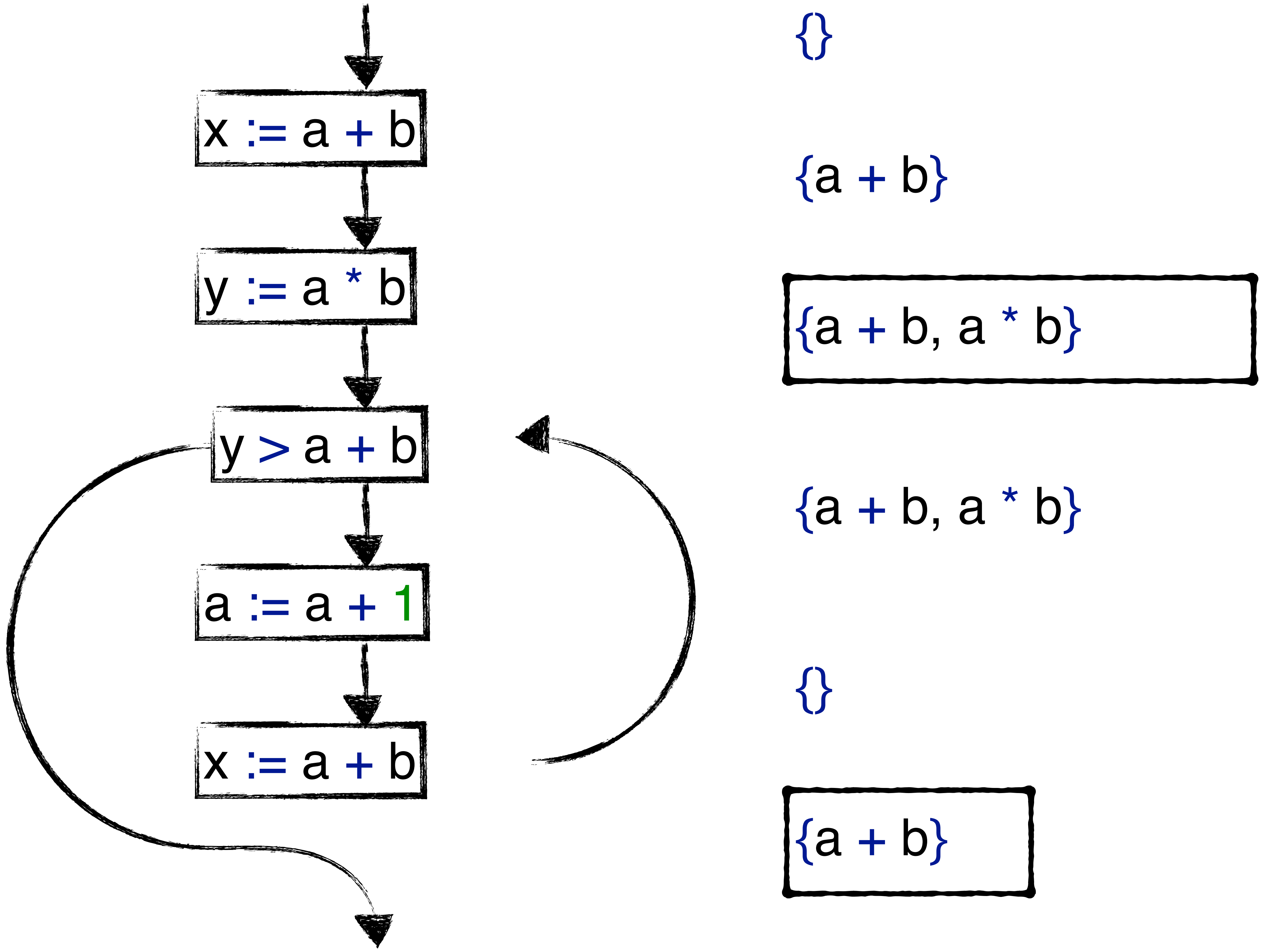

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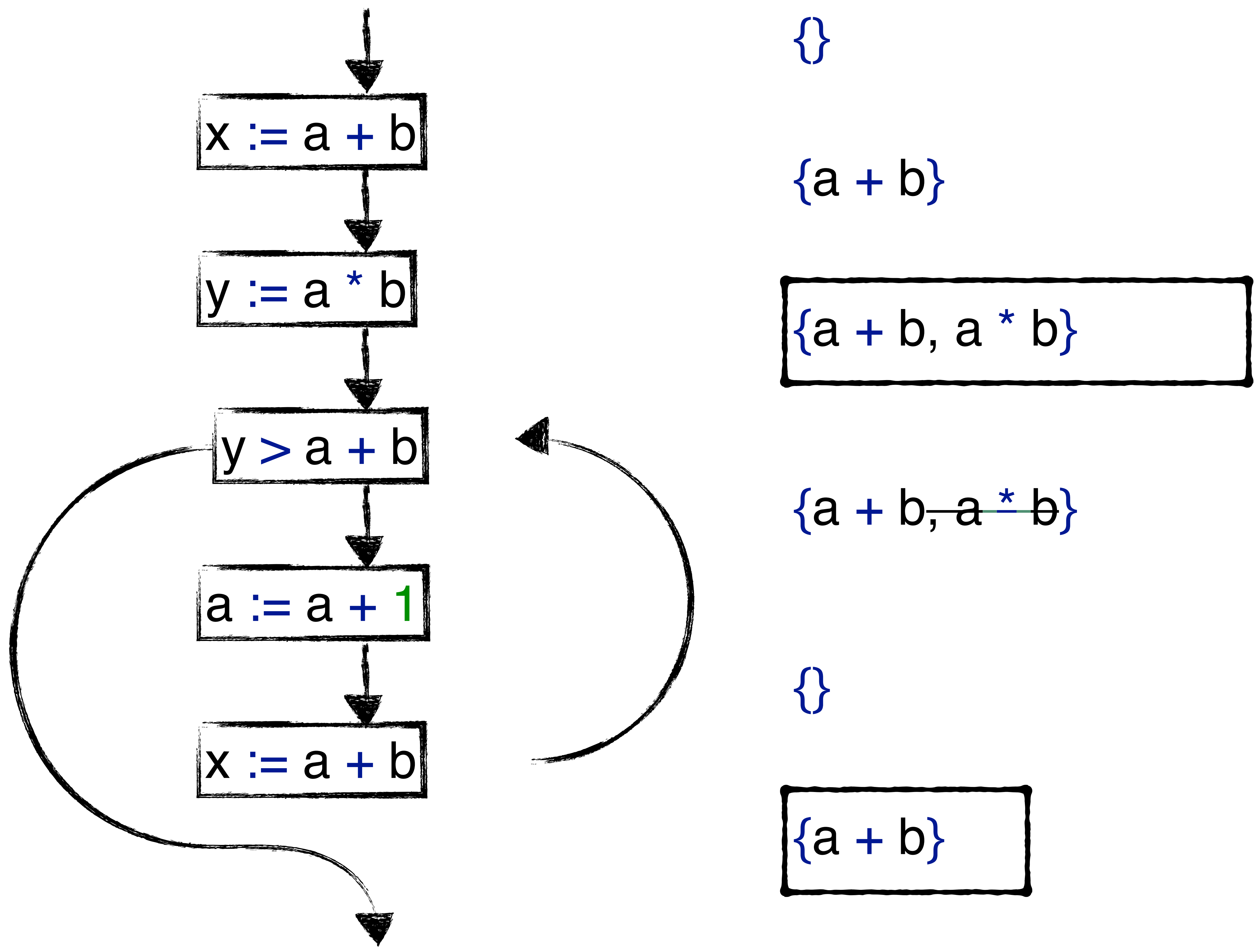

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

Live Variables

"A **variable** is **live** if there exists a path from there to a use of the variable, with no re-definition of the variable on that path. "

```
kill(Assign(var, e1)) :=
  { var }


gen(Assign(var, e1)) :=
  { FV(e1) }

gen(b@BinOp(_, _, _)) :=
  { FV(b) }

gen(u@UnOp(_, _)) :=
  { FV(u) }
```

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

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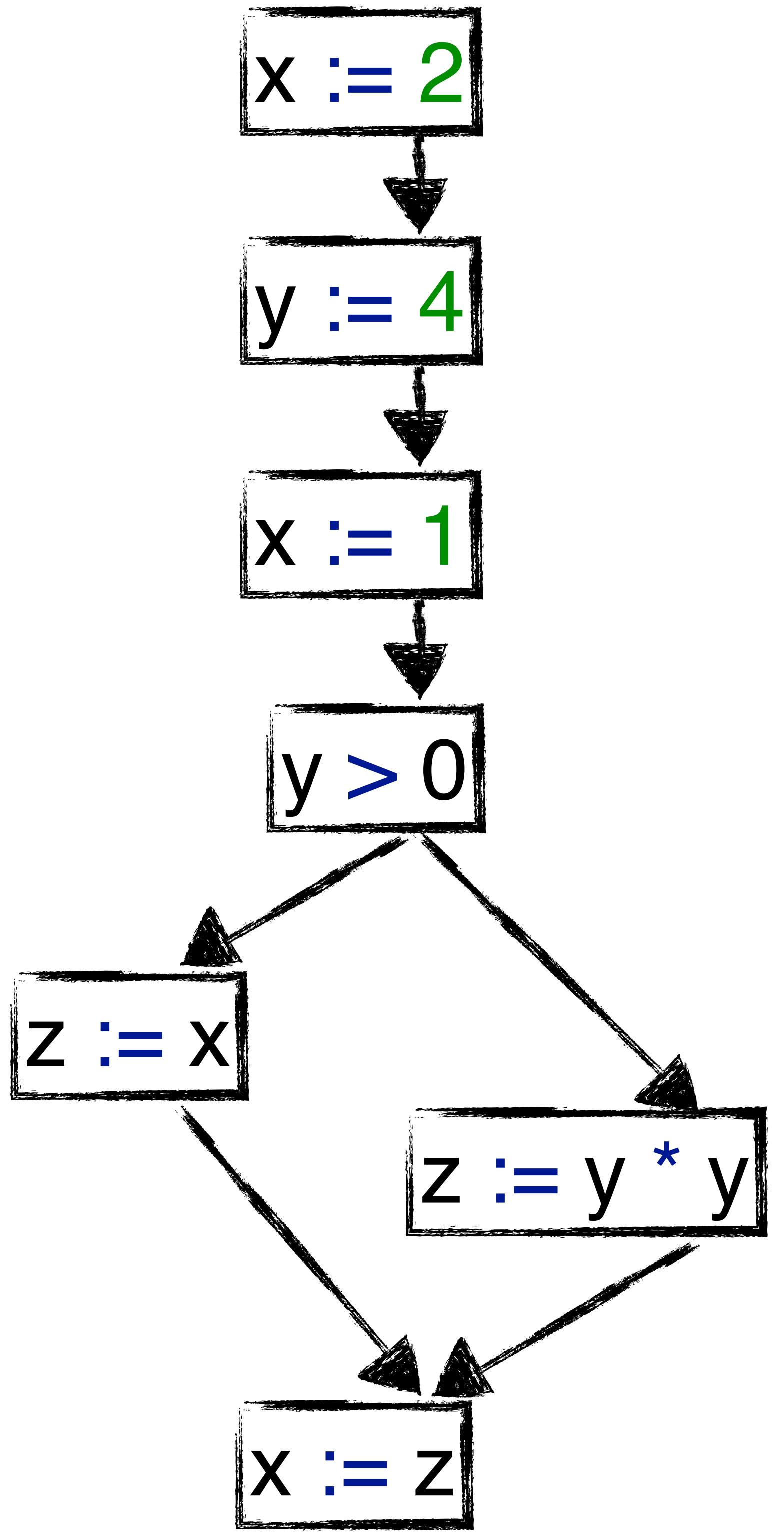

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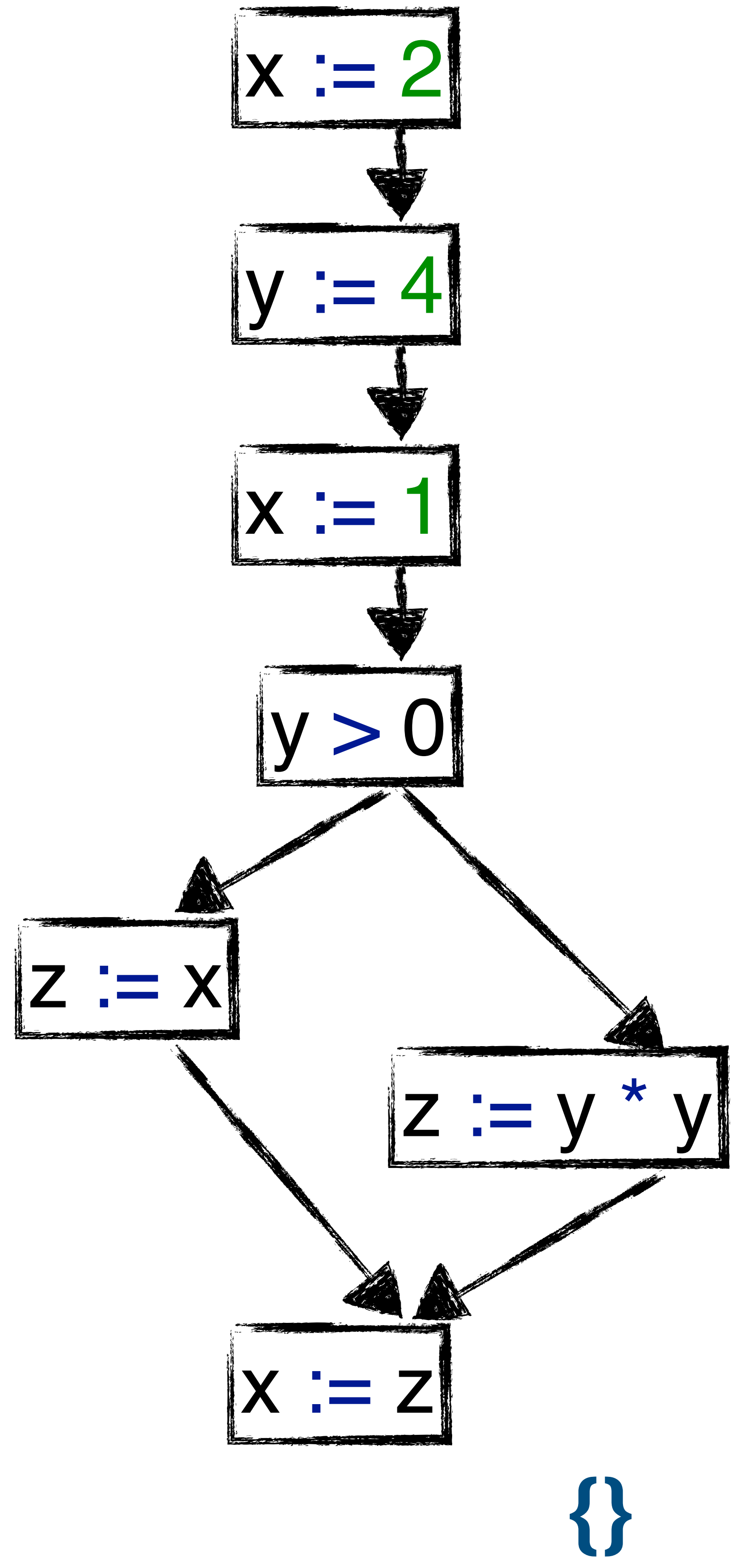

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

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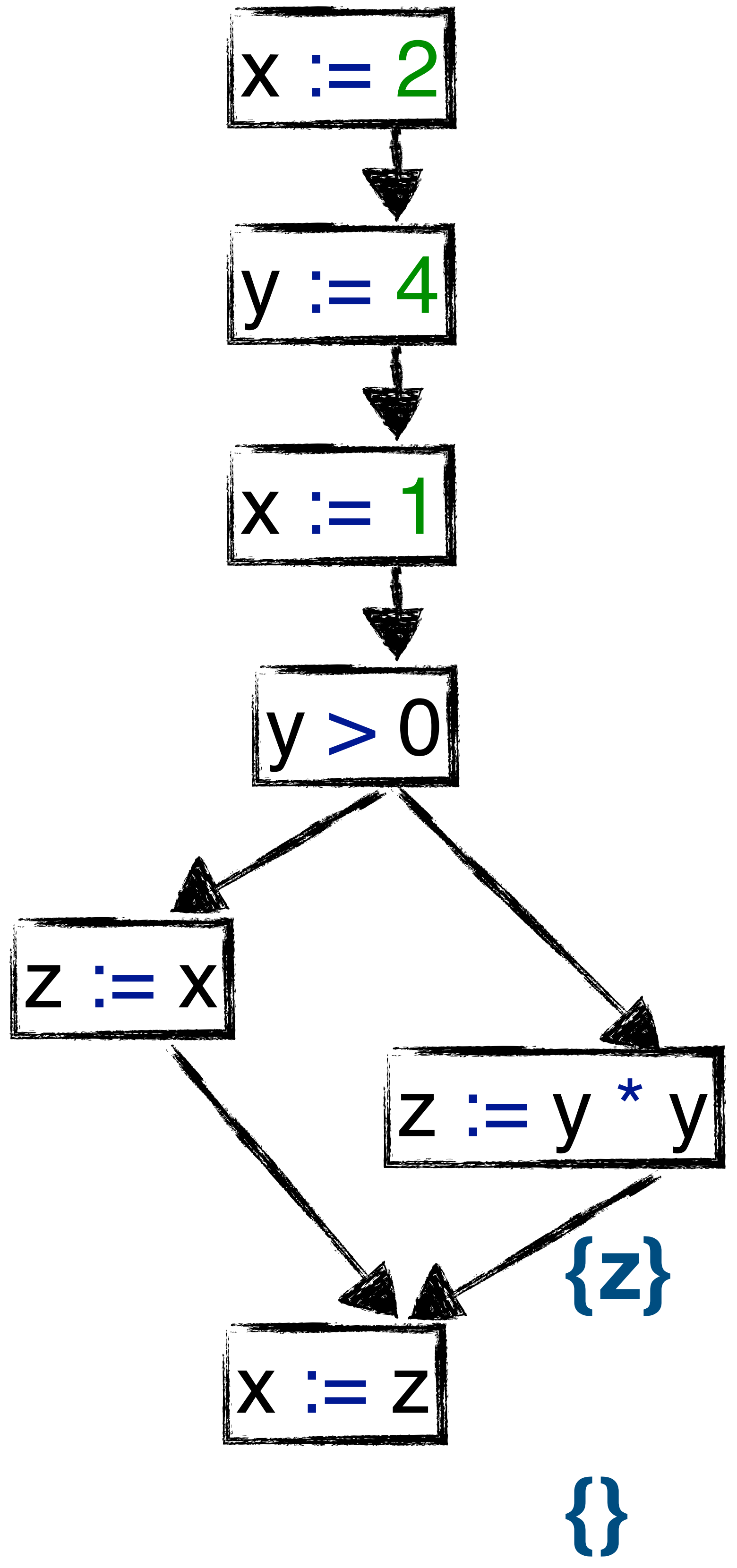

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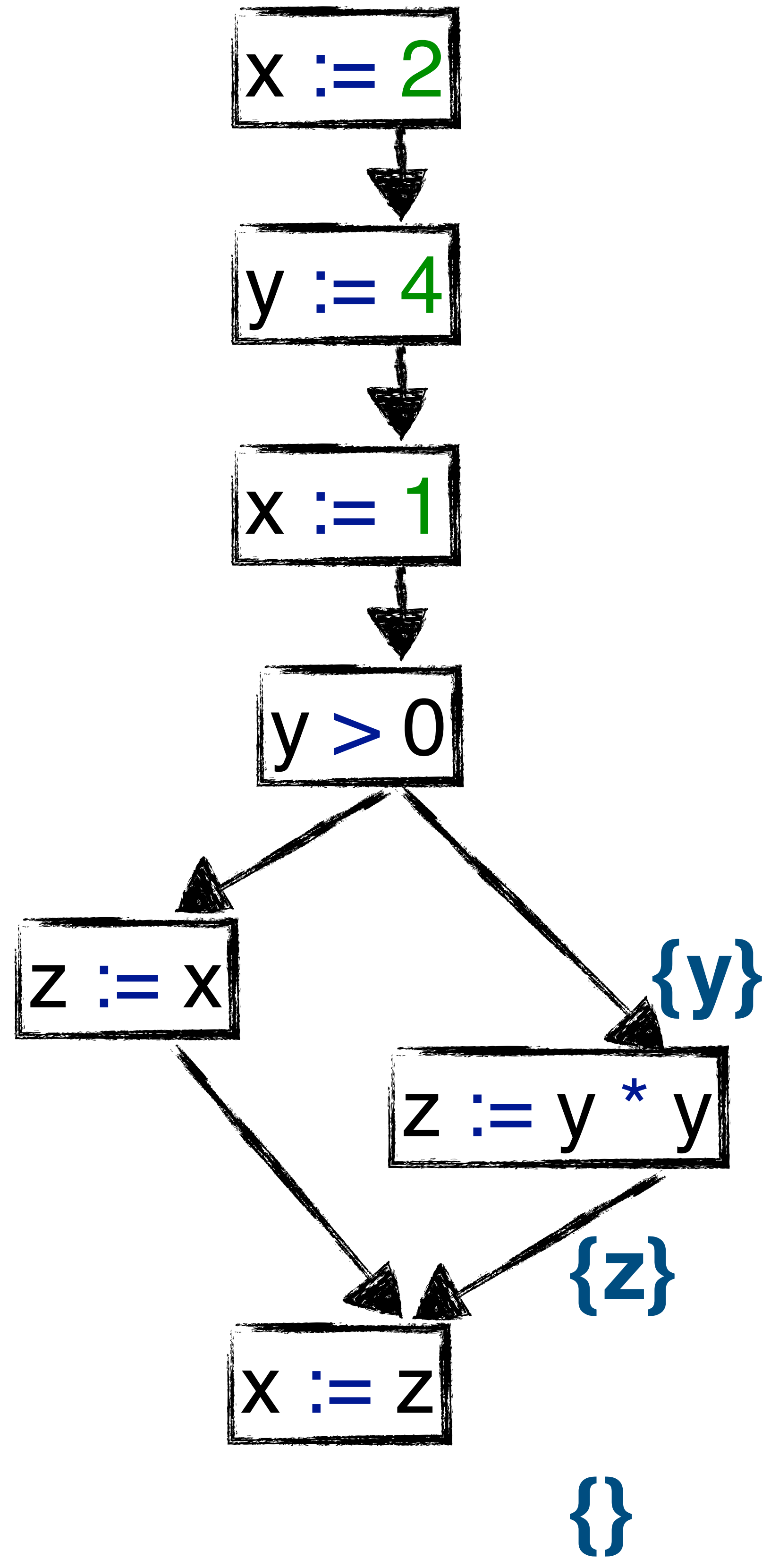

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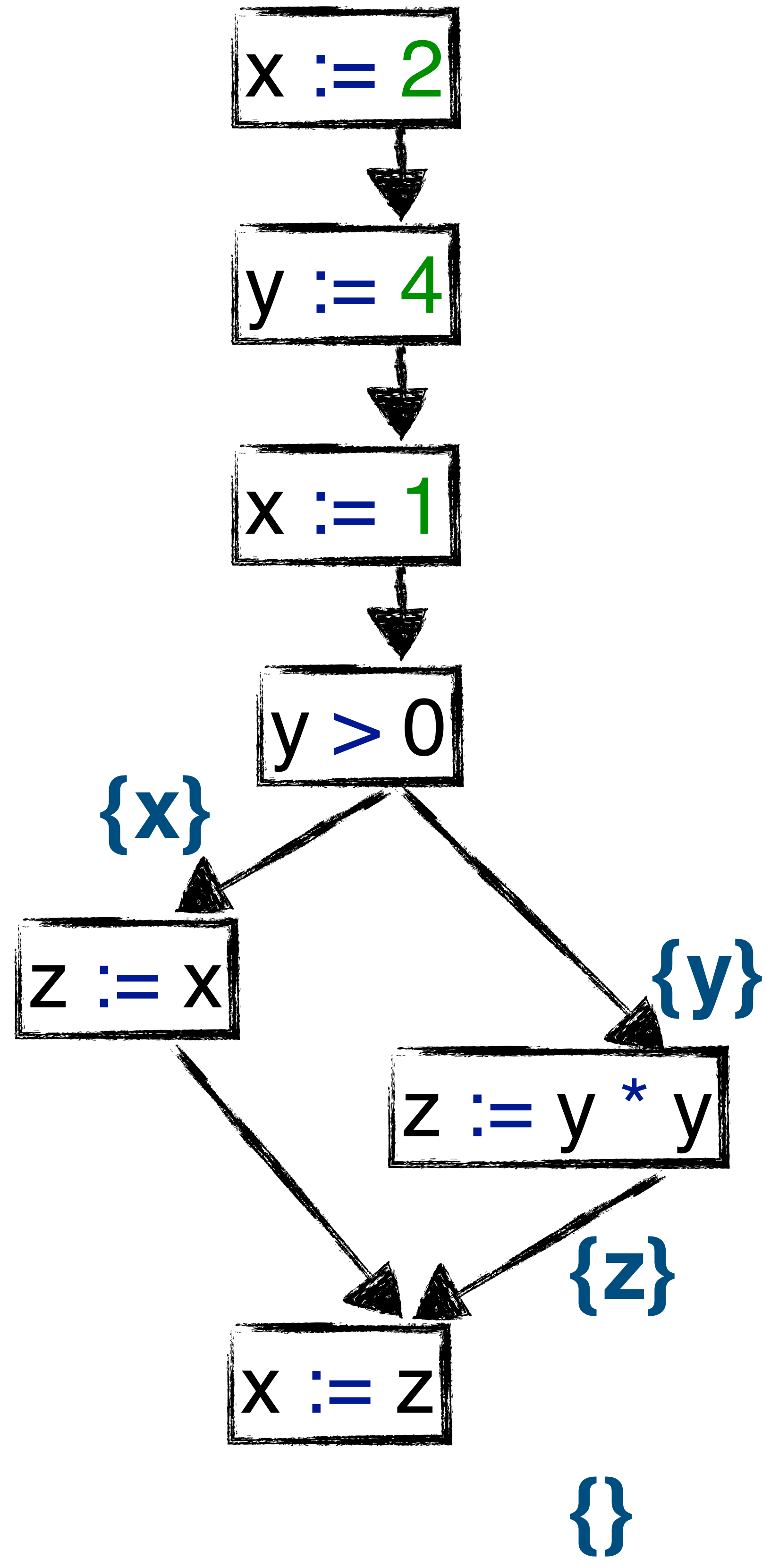

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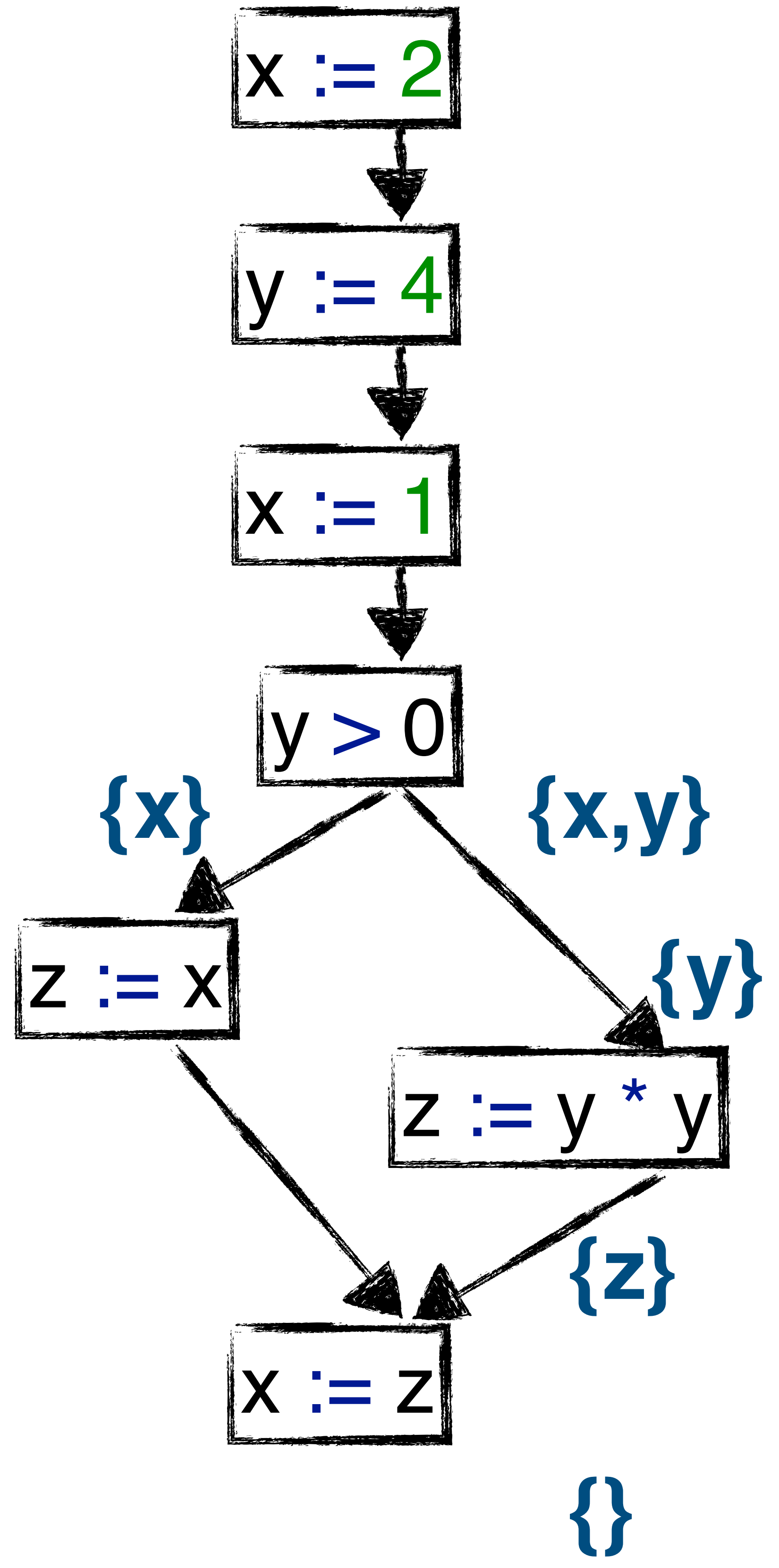

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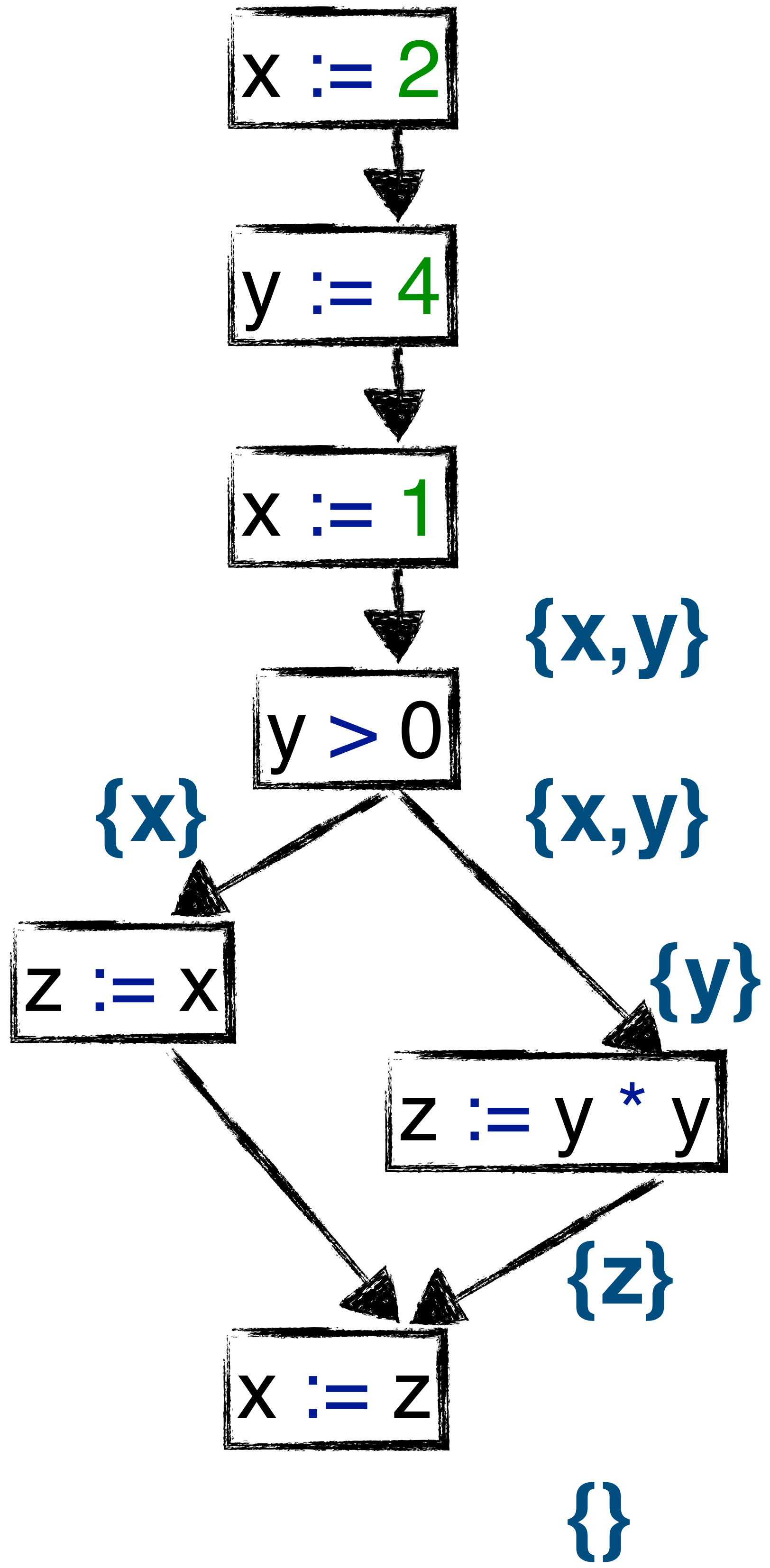

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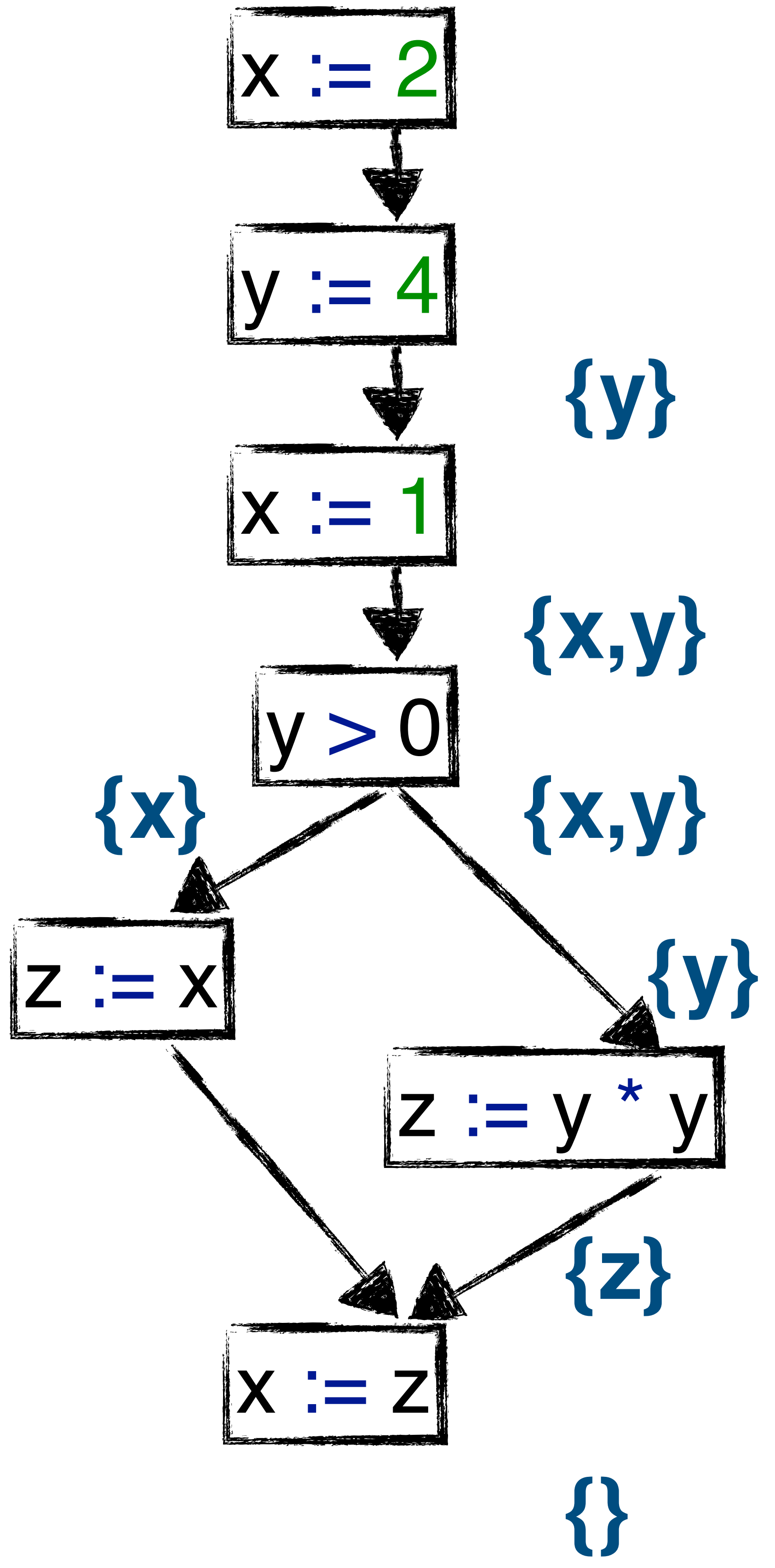

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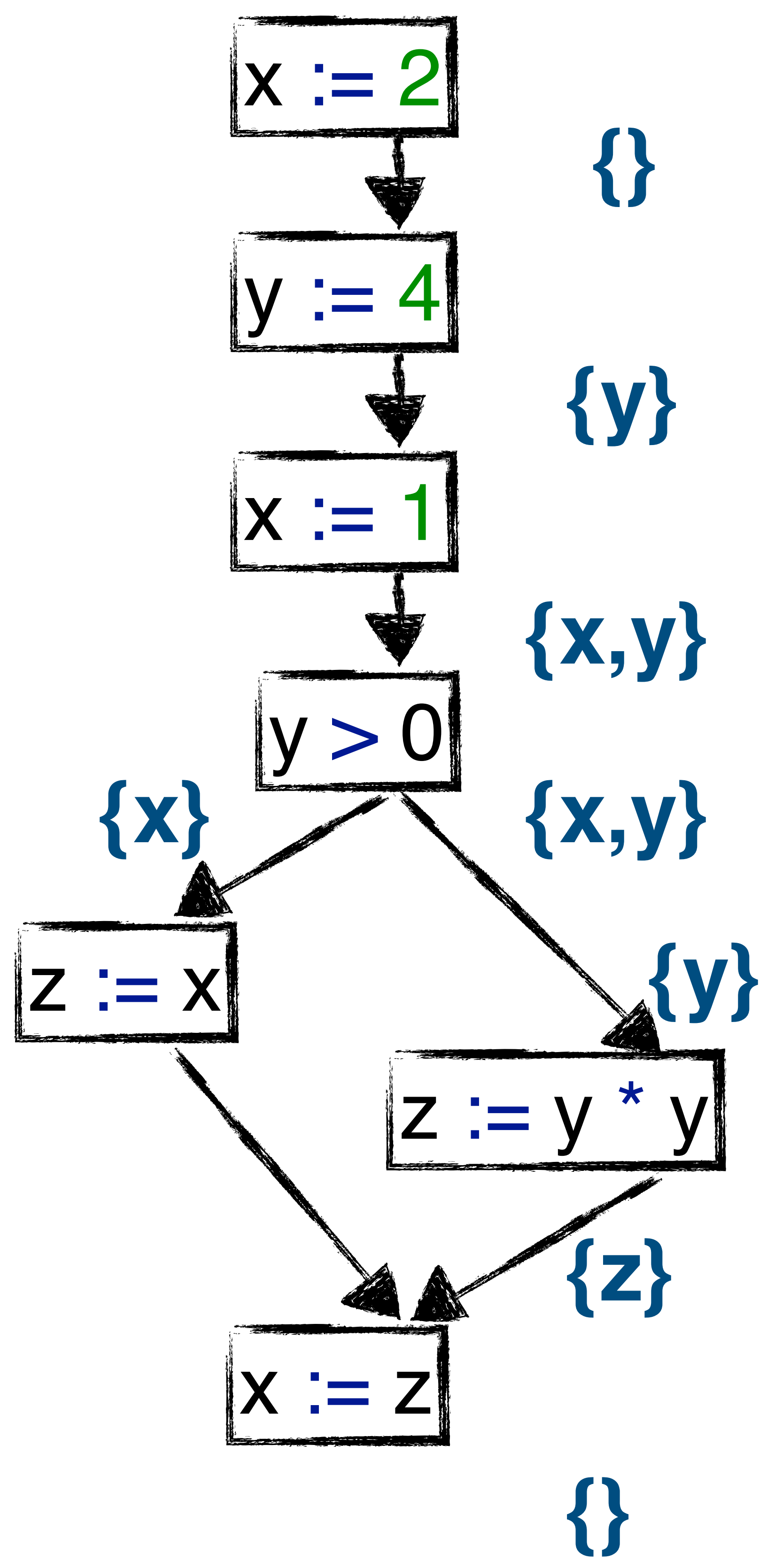

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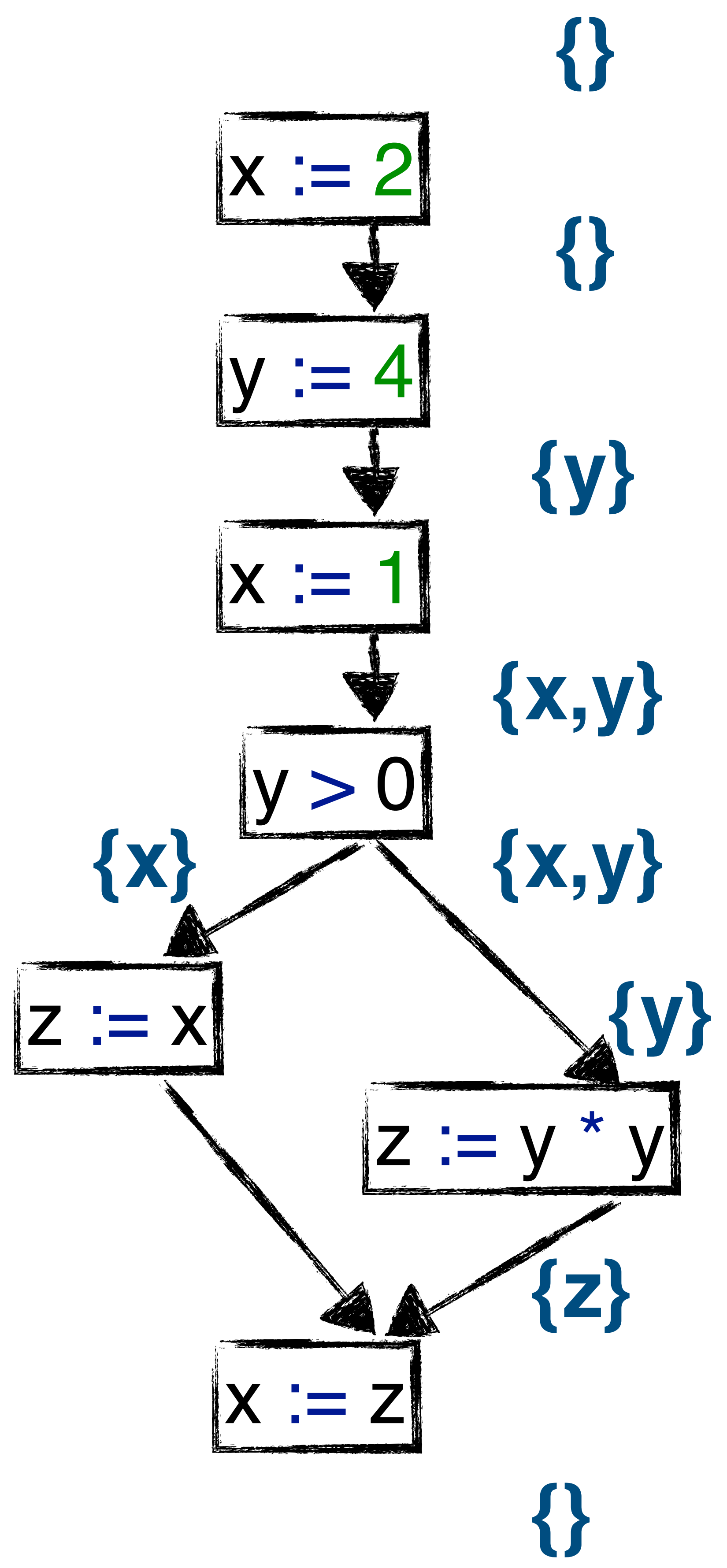

previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

Traditional set based analysis

Sets as analysis information

Traditional set based analysis

Sets as analysis information

Kill and gen sets per control node type

Traditional set based analysis

Sets as analysis information

Kill and gen sets per control node type

```
–previousSet \ kill(currentNode) ∪ gen(currentNode)
```

Traditional set based analysis

Sets as analysis information

Kill and gen sets per control node type

- `previousSet \ kill(currentNode) ∪ gen(currentNode)`

Can propagate either forward or backward

Traditional set based analysis

Sets as analysis information

Kill and gen sets per control node type

```
—previousSet \ kill(currentNode) ∪ gen(currentNode)
```

Can propagate either forward or backward

Can merge information with either union or intersection

Traditional set based analysis

Sets as analysis information

Kill and gen sets per control node type

```
−previousSet \ kill(currentNode) ∪ gen(currentNode)
```

Can propagate either forward or backward

Can merge information with either union or intersection

- Respectively called may and must analyses

Beyond Sets

Constant propagation and folding

```
let
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Kill/gen doesn't work here

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But what if we keep multiple values?

- Analysing loops may not terminate

Example: Non-termination

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a ↦ 0,1; b ↦ 1   a ↦ 0,1,2; b ↦ 1

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- Variables bound to either a particular constant or a marker for non-constants

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The transfer functions per control node

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- Needs to propagate markers when found

Monotone Frameworks

Termination

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Data-Flow Analysis needs fixpoint computation

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Data-Flow Analysis needs fixpoint computation

- Because of loops

Lattice Theory

A set X is totally ordered under ≤ if for a, b, c ∈ X

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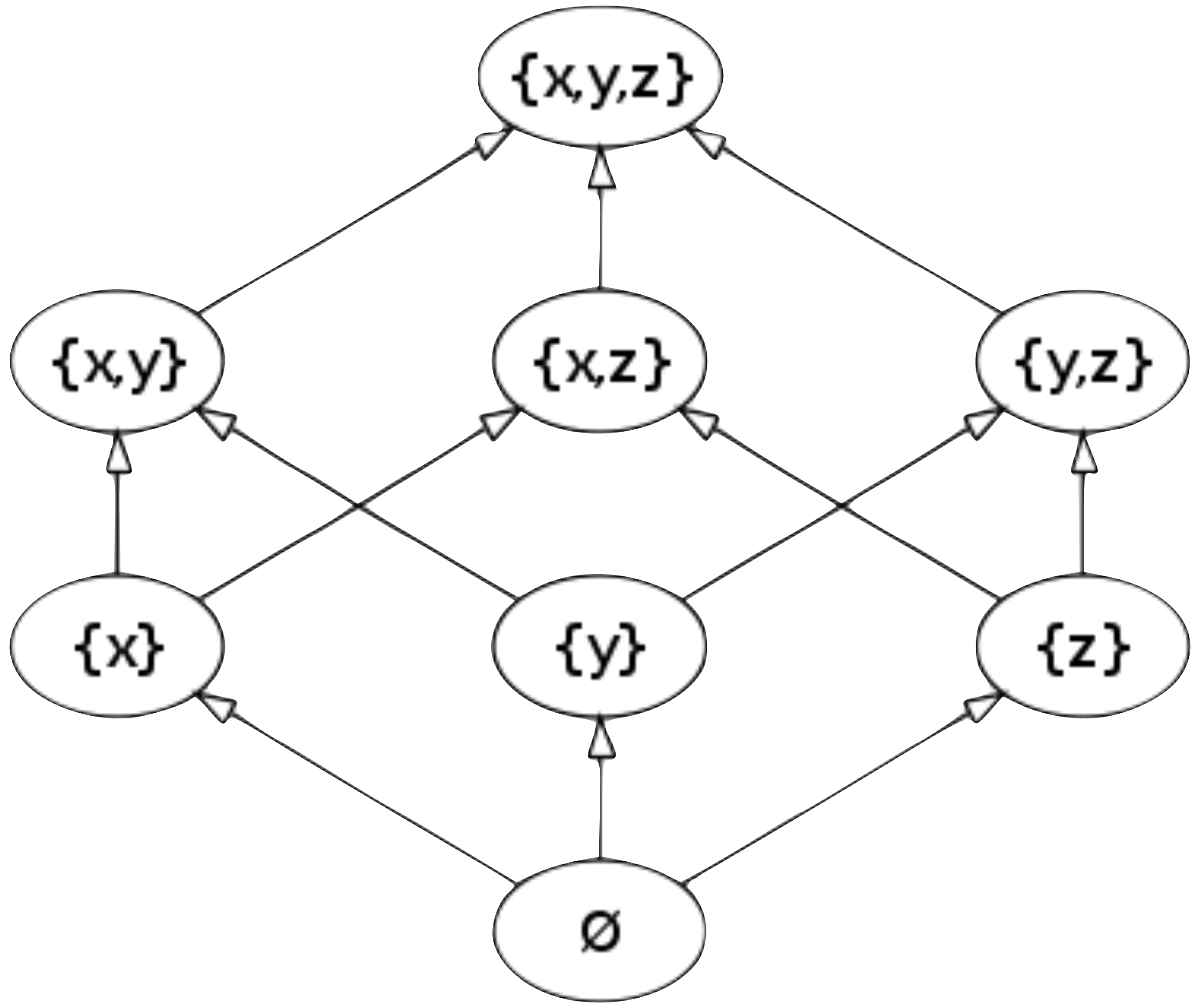
- e.g. subset inclusion:



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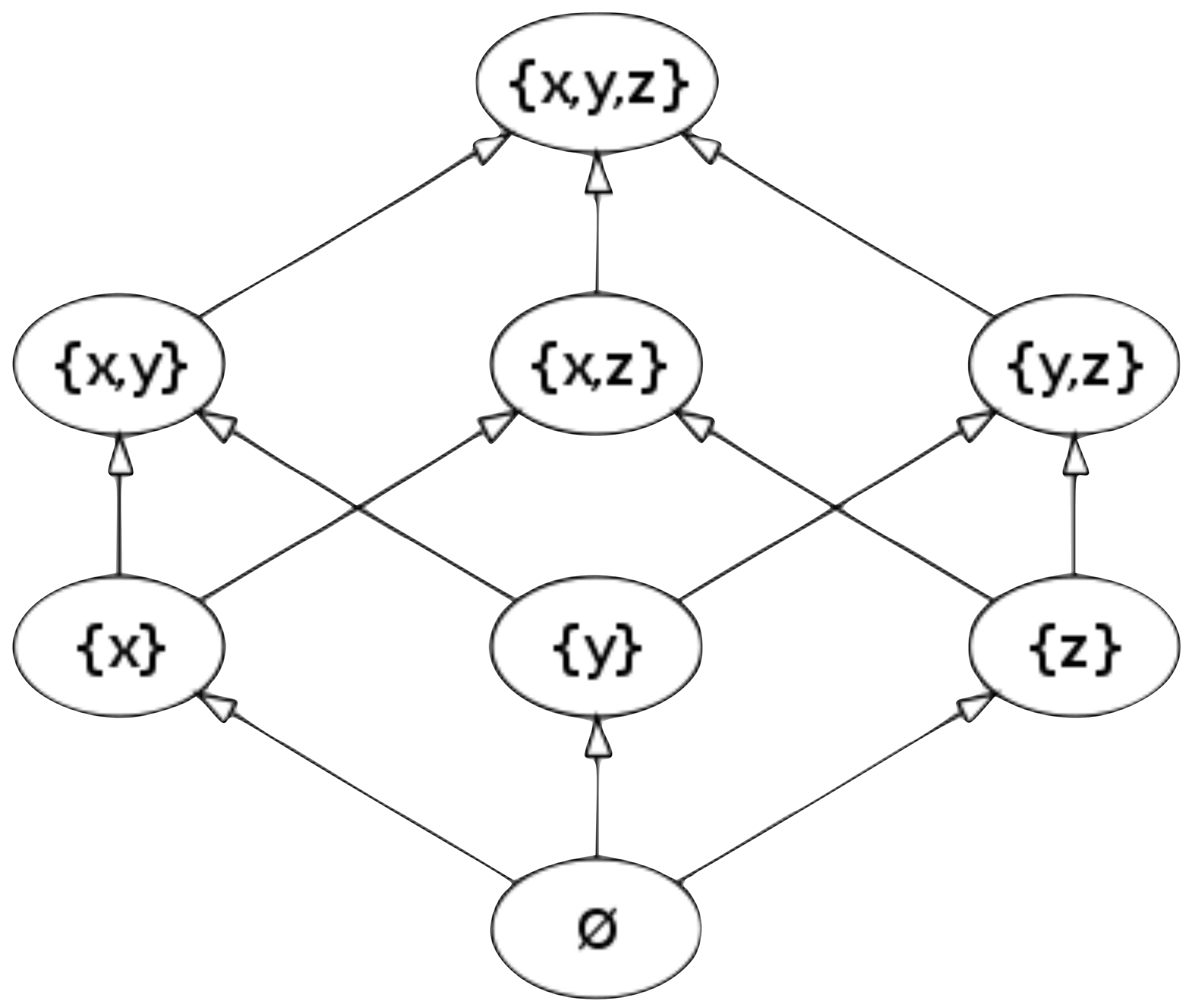

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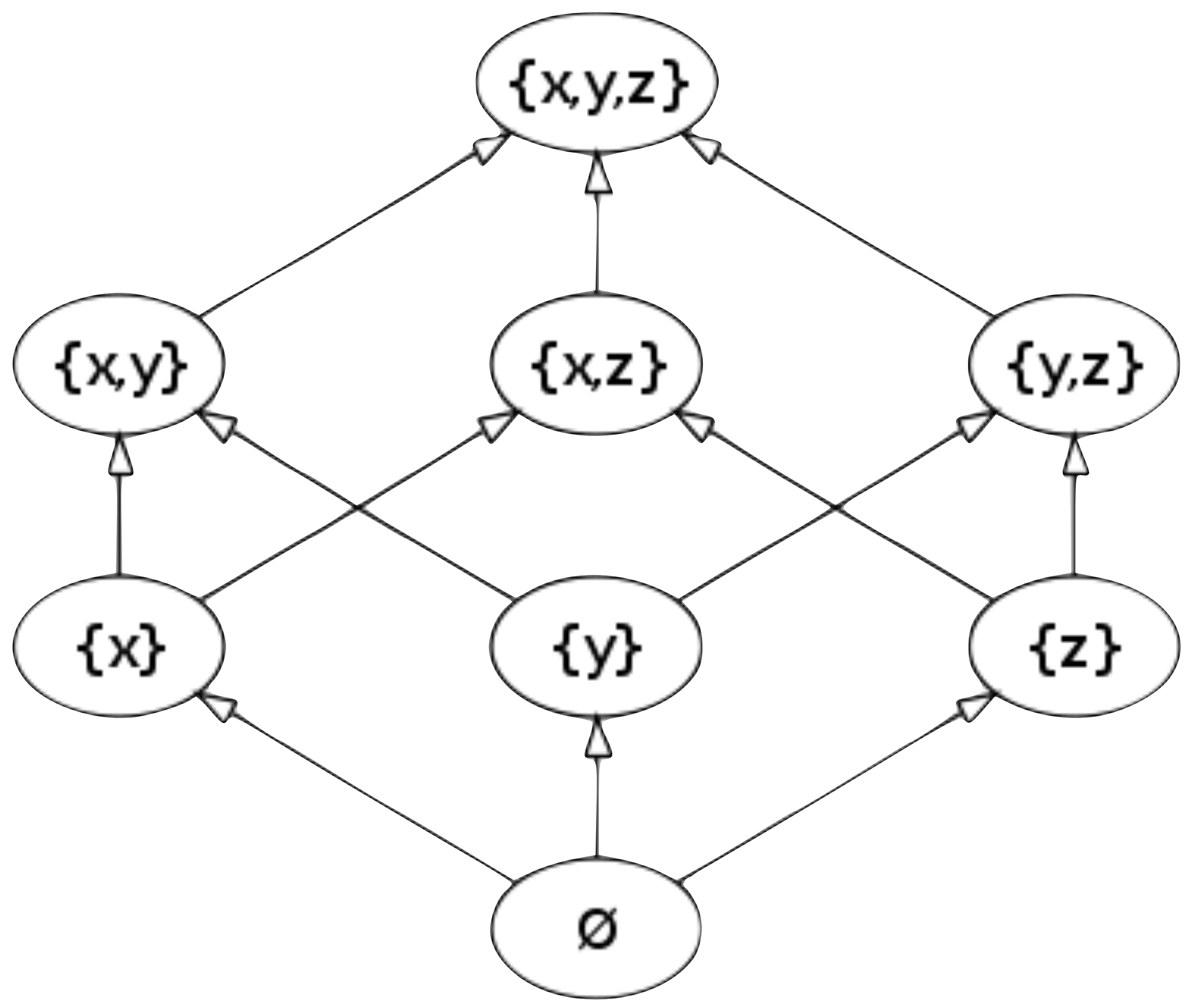

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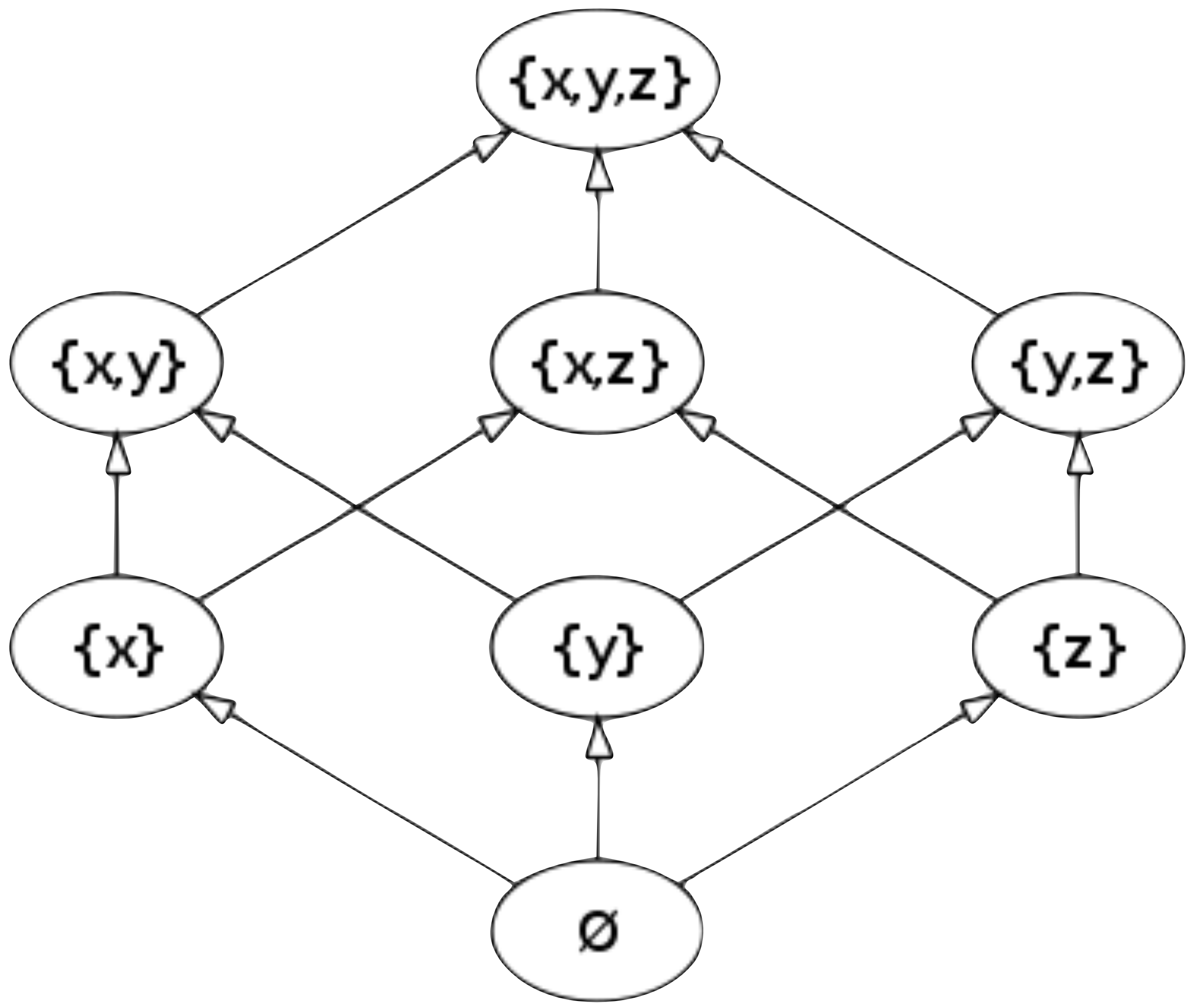

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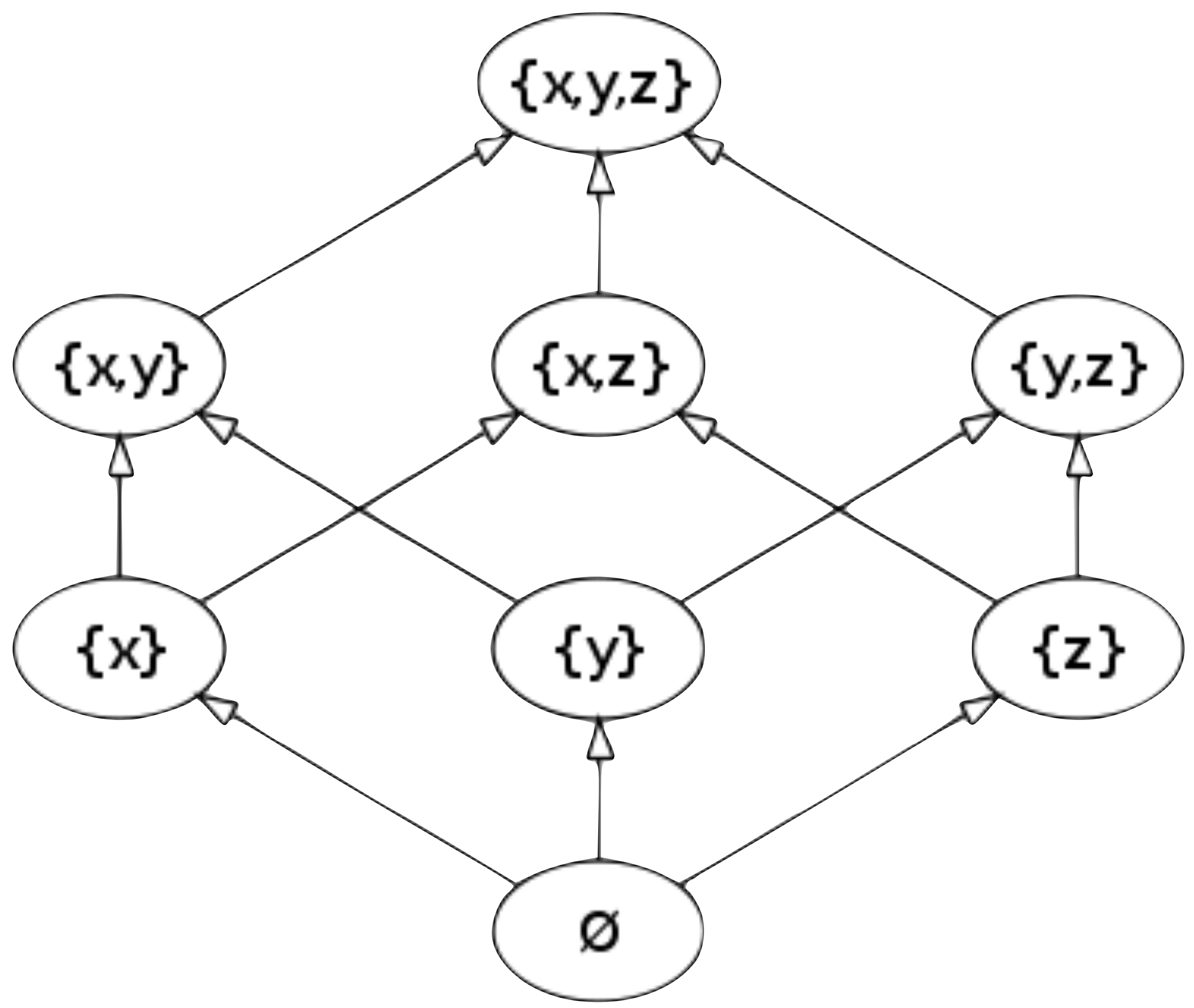

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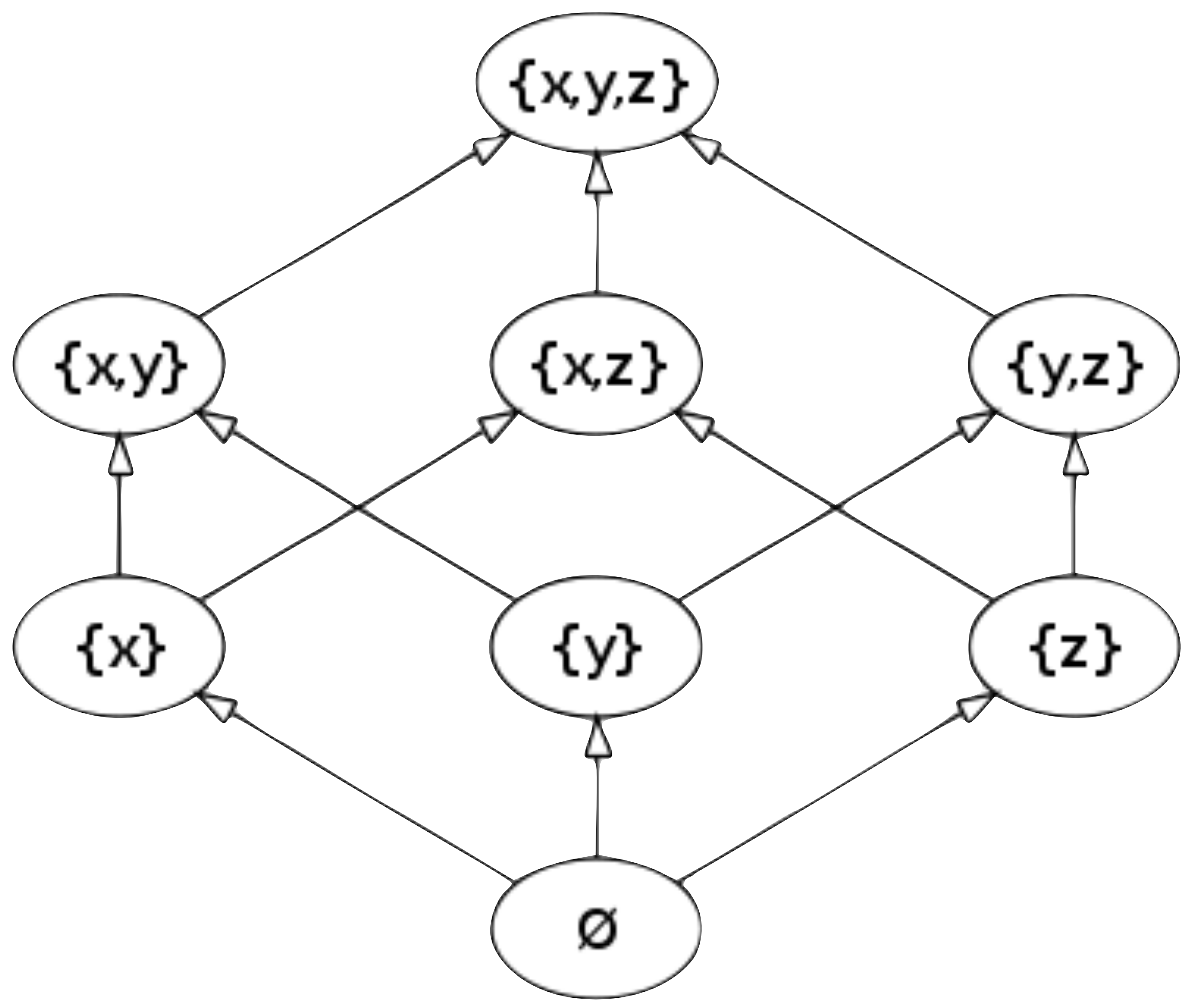

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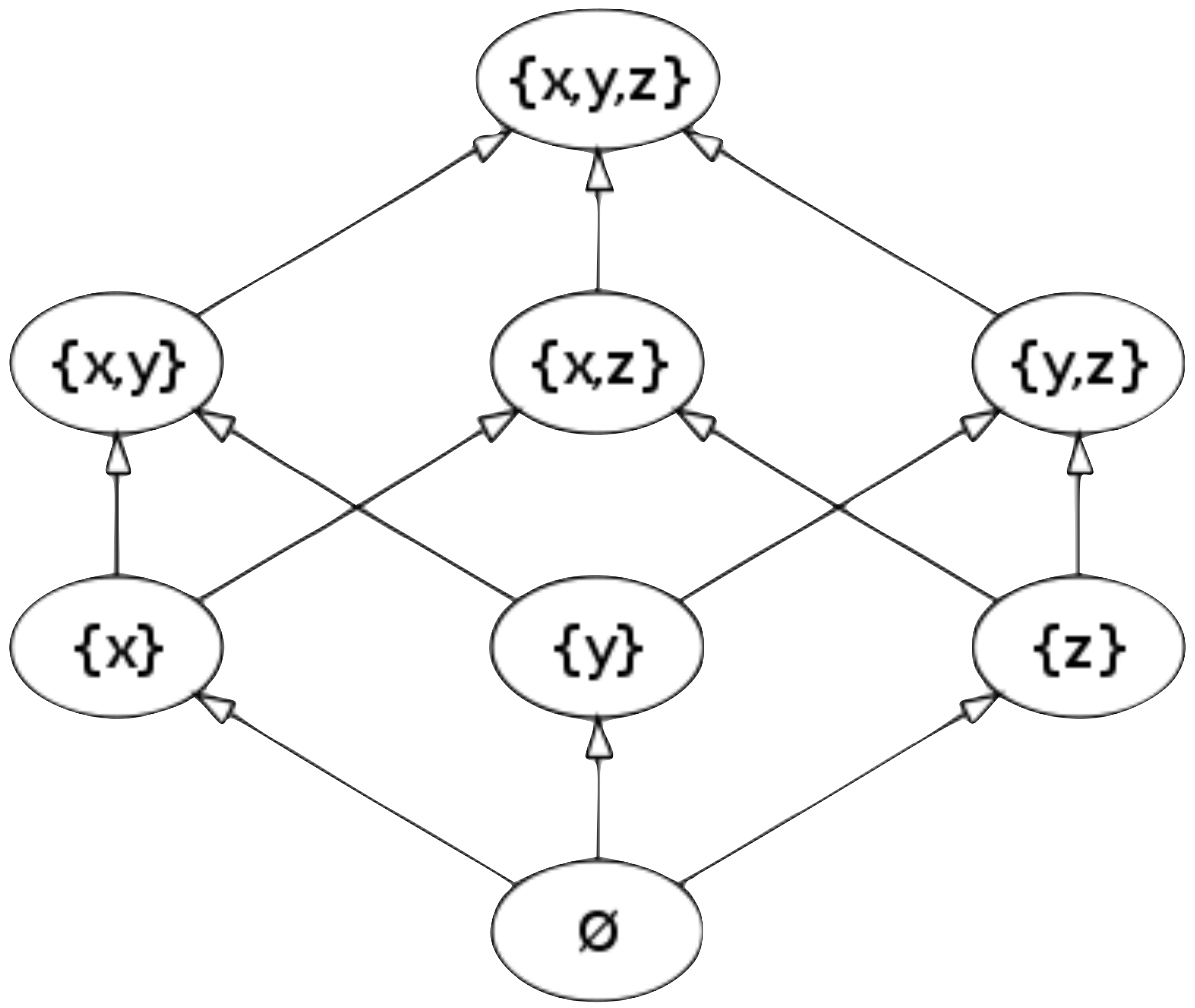

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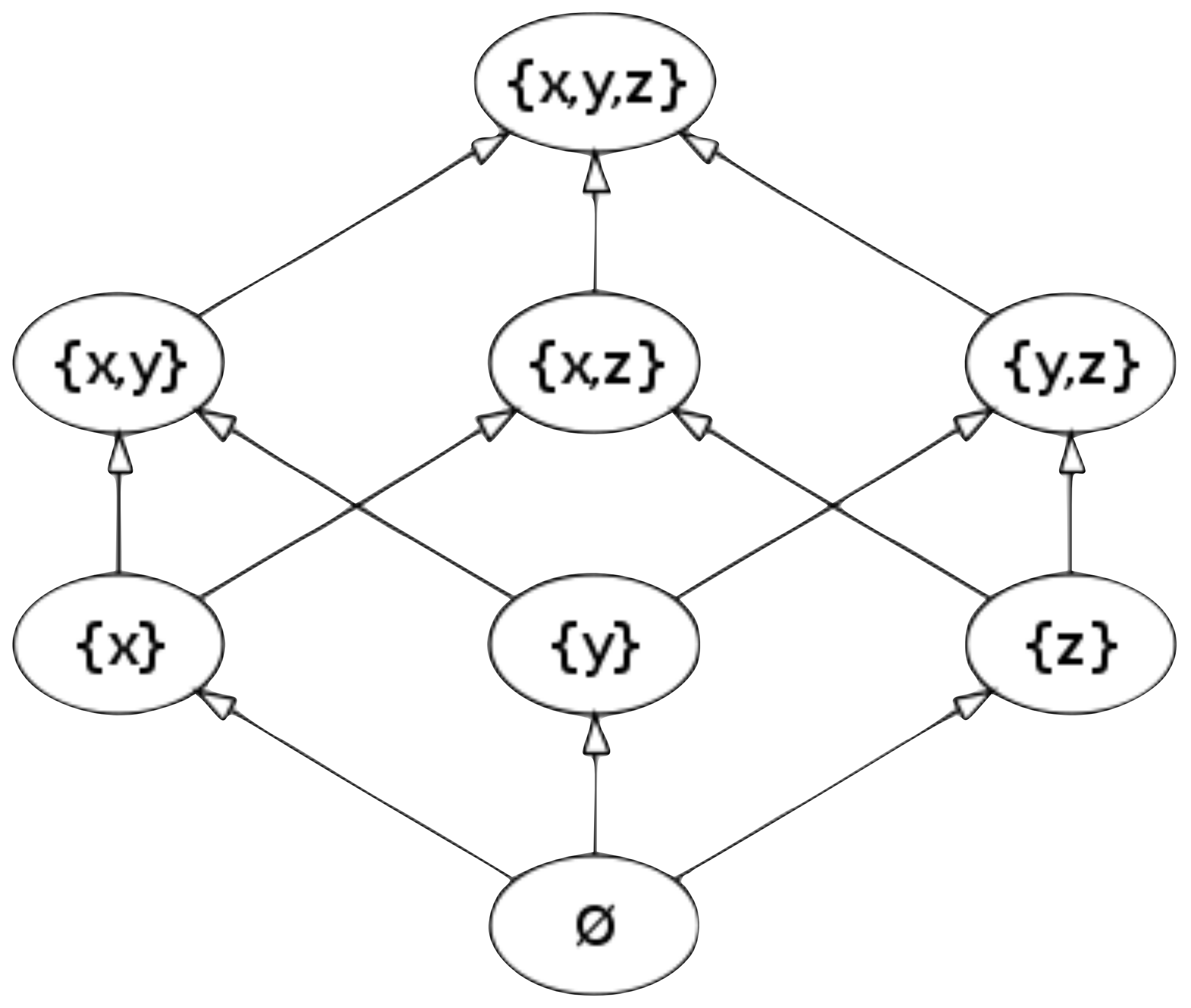

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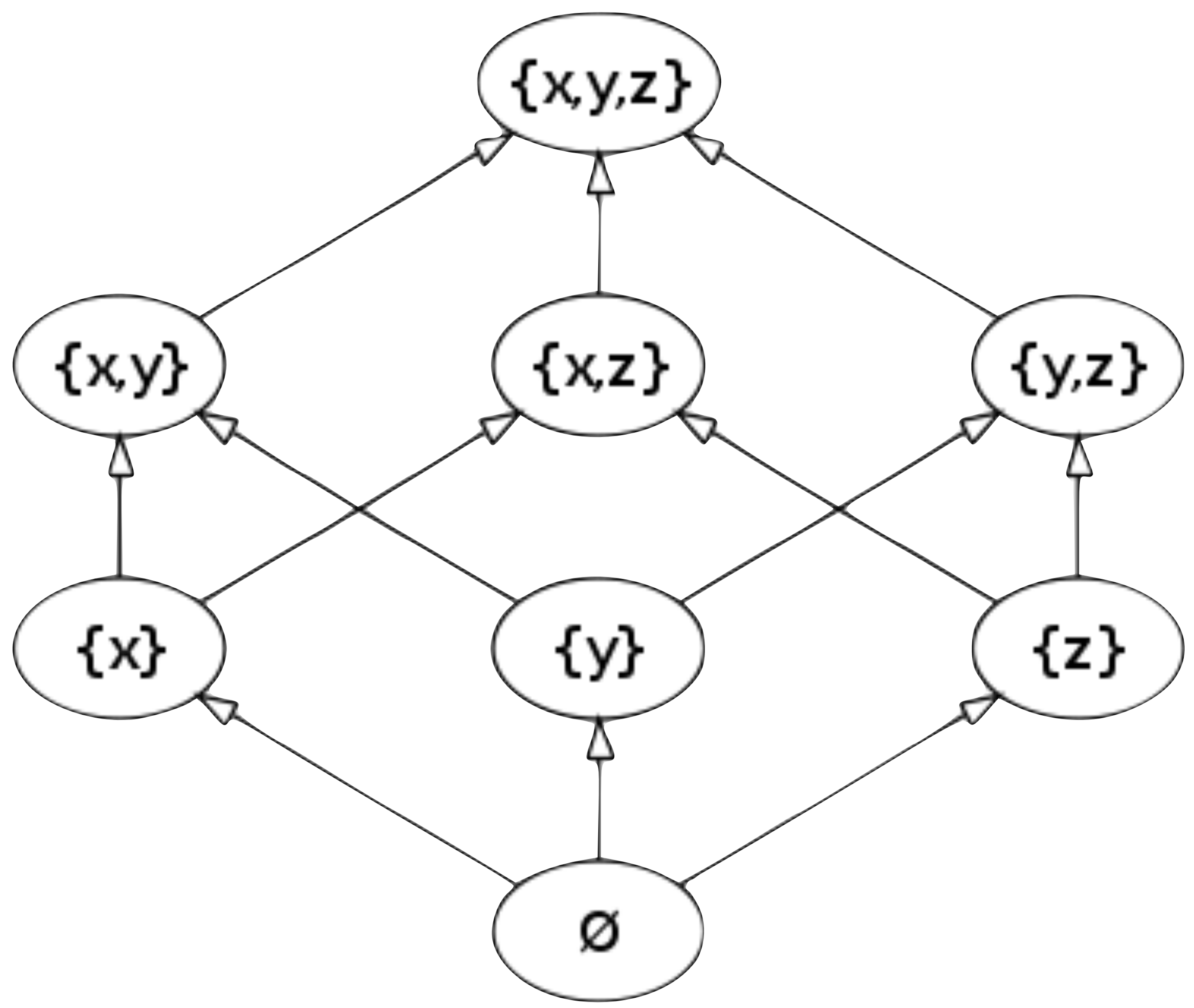

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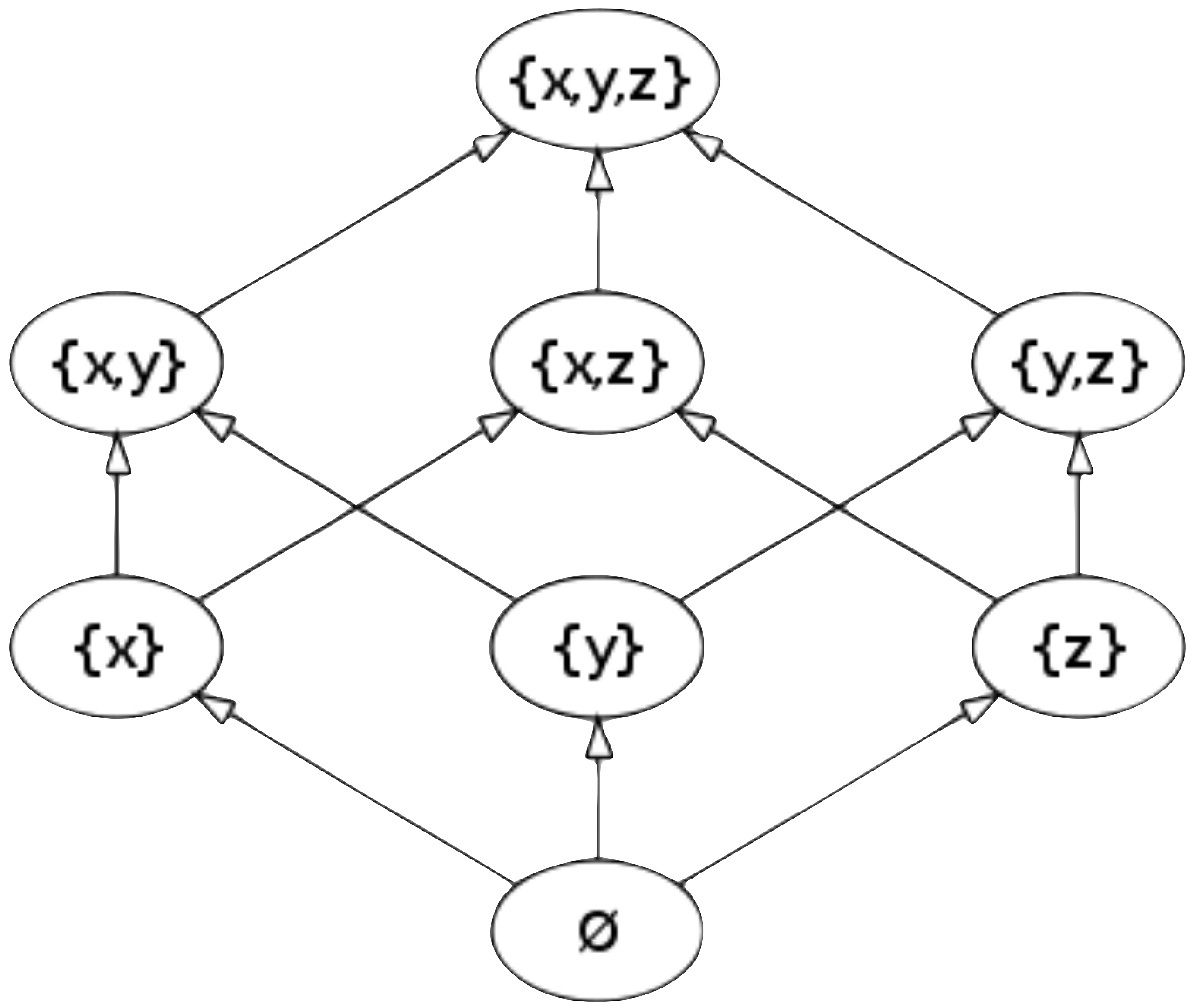

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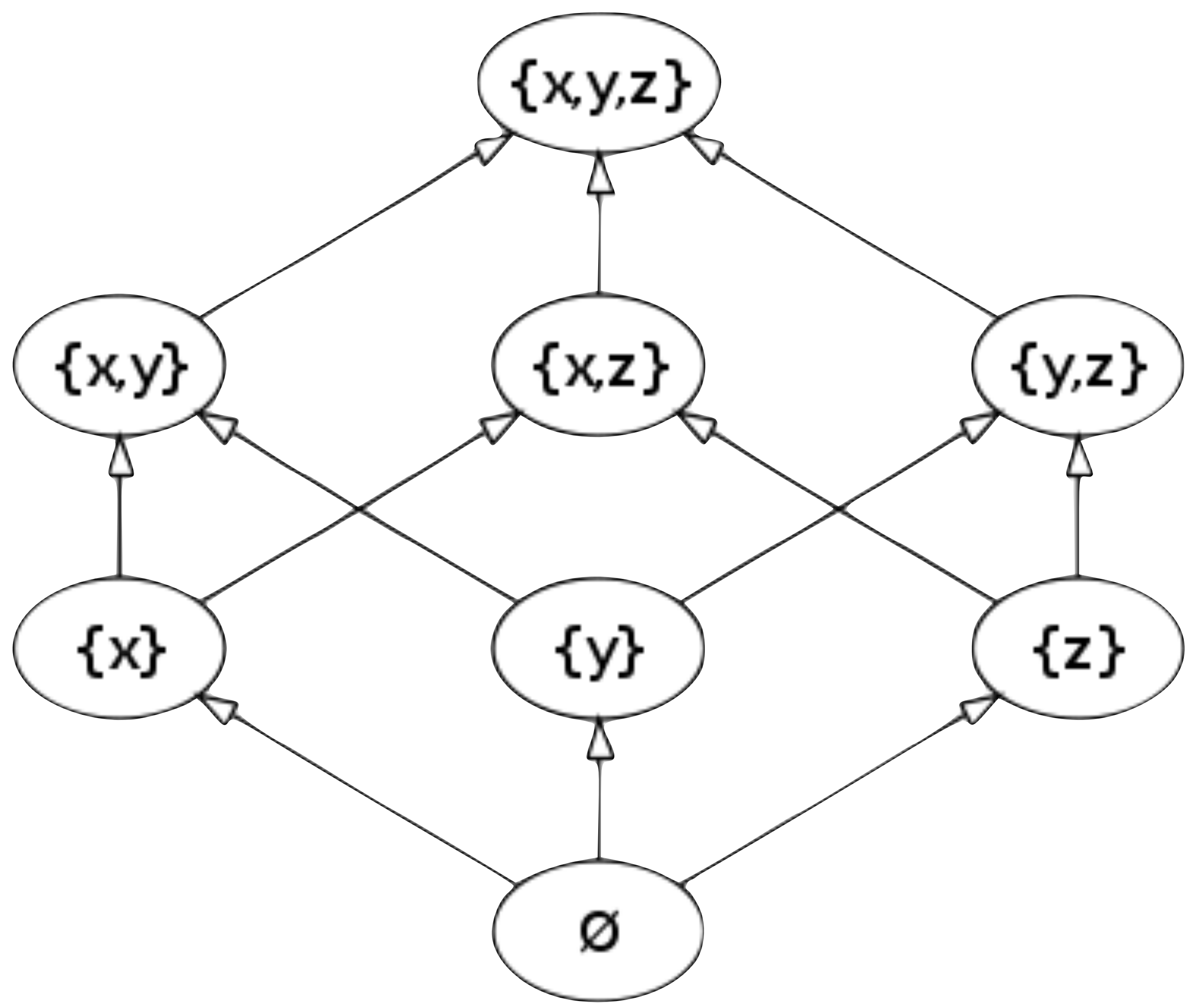

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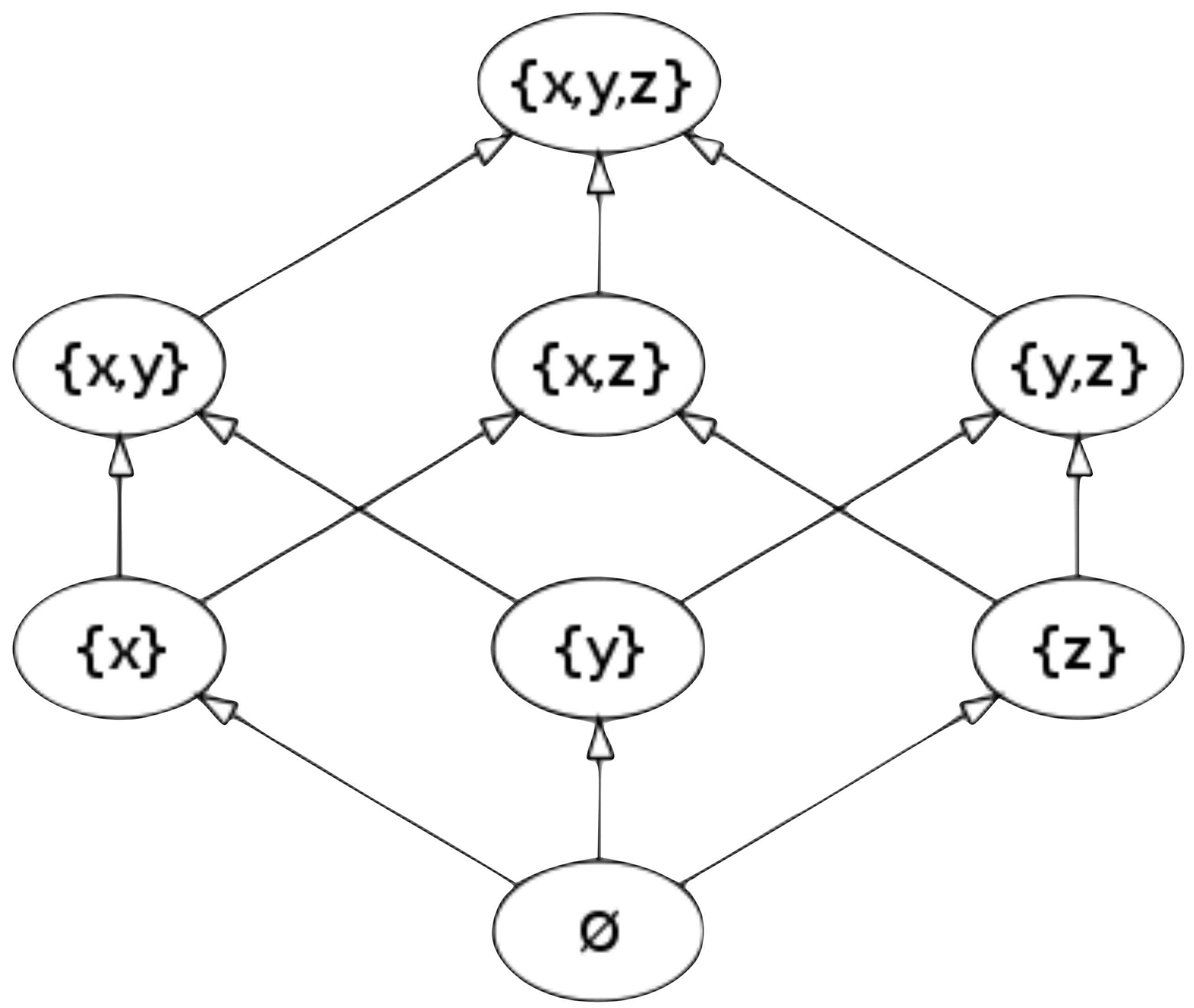

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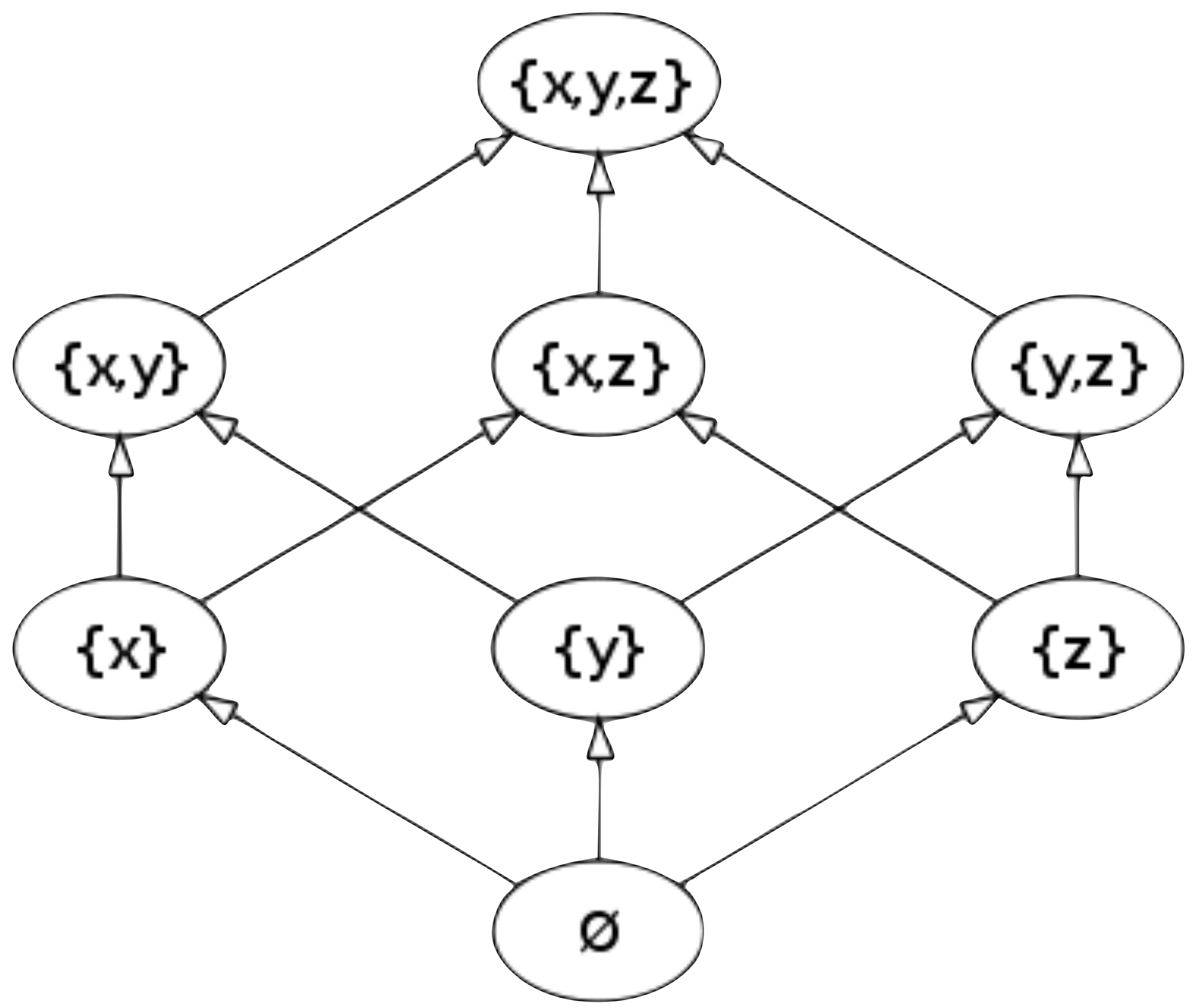

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- These are $\top$ and $\bot$ respectively

Lattices for Data-Flow Analysis

Consider $\top$ as the coarsest approximation

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- It is the most information preserving combination of information

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Transfer functions should be monotone increasing

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General interval analysis has an infinite lattice

- $\top = [-\infty,\infty]$
- If a loop adds a finite number to a variable, you never get to ∞

Recap

Recap

An analysis consists of

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Recap

An analysis consists of

- The type of the analysis information, **and the lattice instance for that type**
- The *transfer functions* that express the ‘effect’ of a control node
  - **These should be monotone with respect to the lattice**
- The initial analysis information

Executing Monotone Frameworks

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Great formal model for reasoning

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But how to execute?

Framework Overview

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Control-flow graph

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Initial data-flow information for start node

Transfer function $f : (L \rightarrow L)$ per control-flow graph node

- Denotes the data-flow effect of the CFG node

Naive Approach

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start_node.value = initial_value
walk(start_node)

function walk(node) =
  for next in node.successors:
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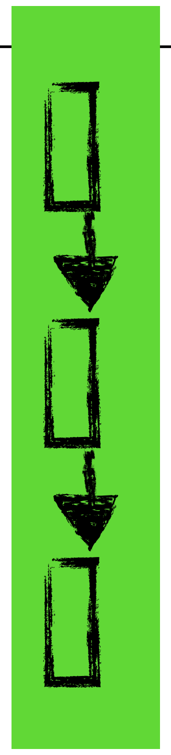

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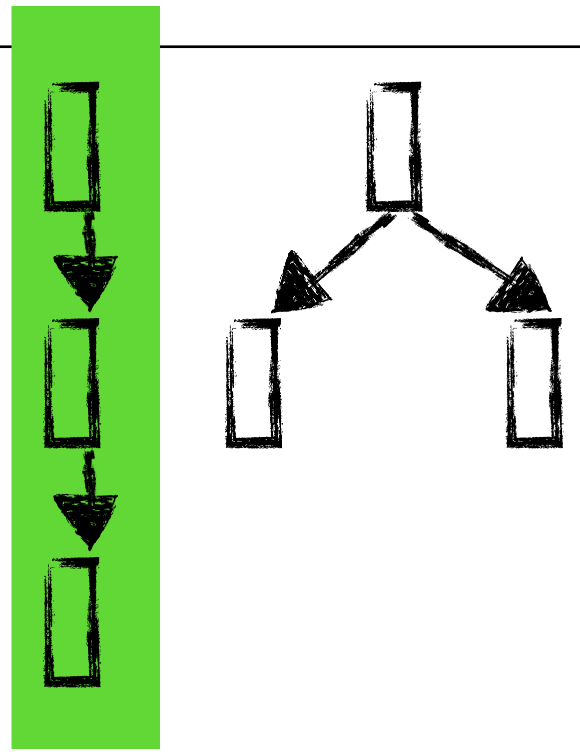

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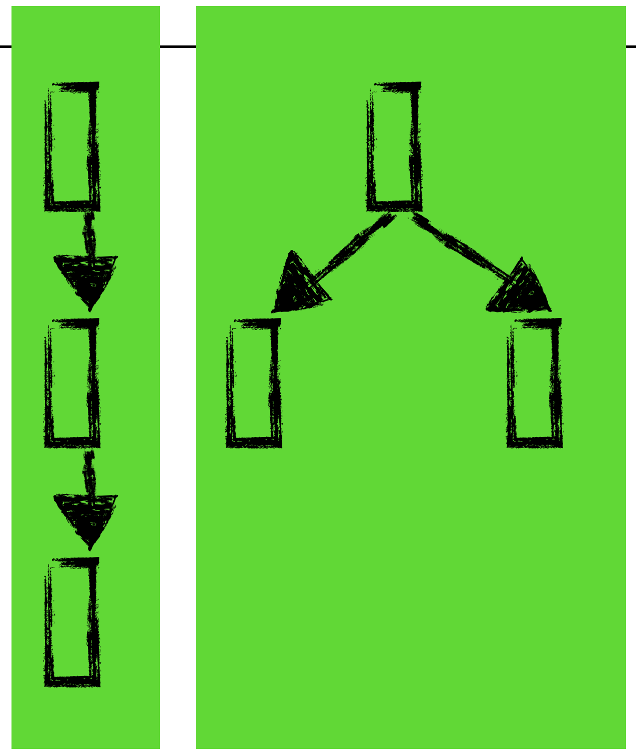

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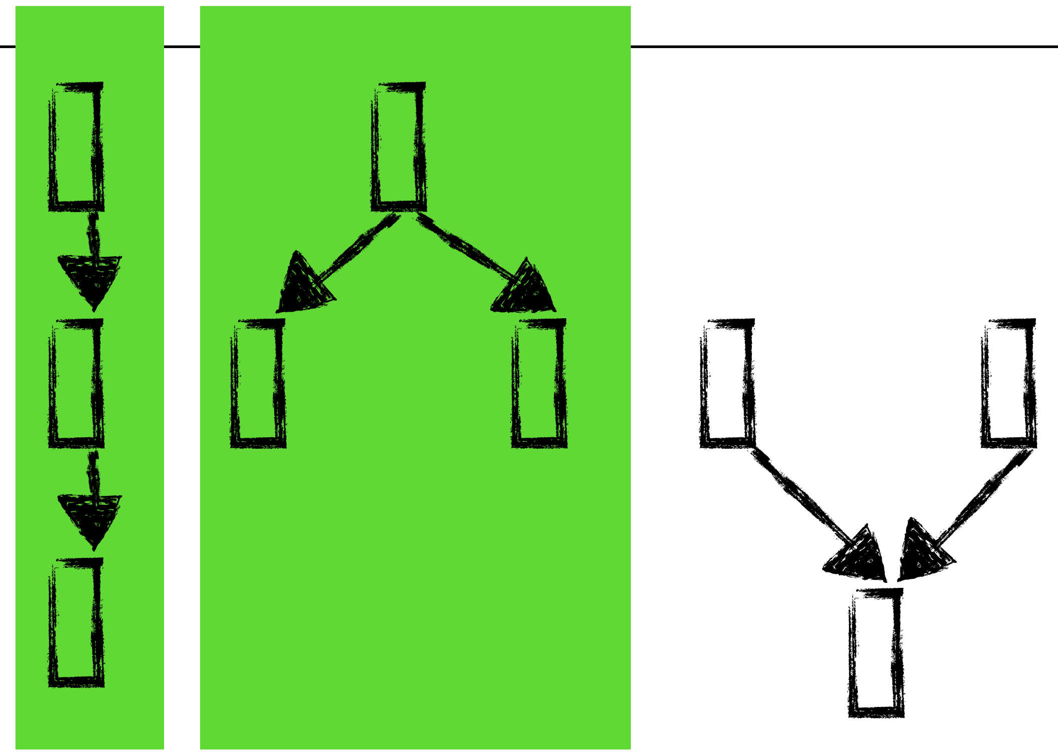

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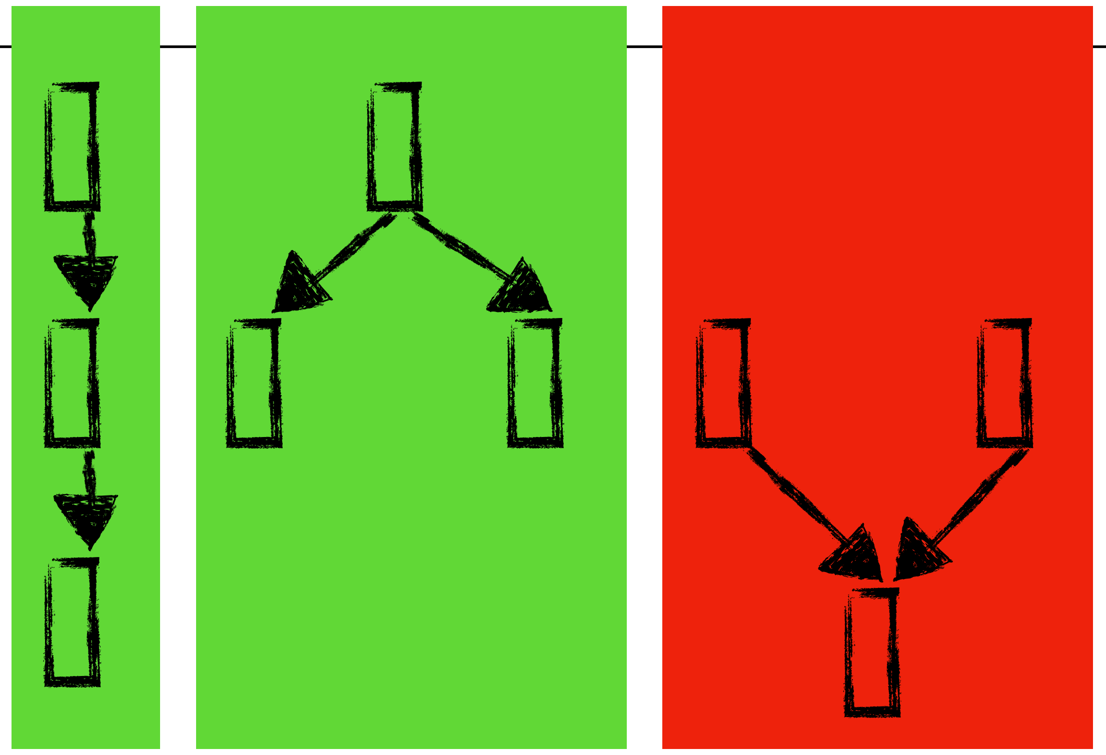

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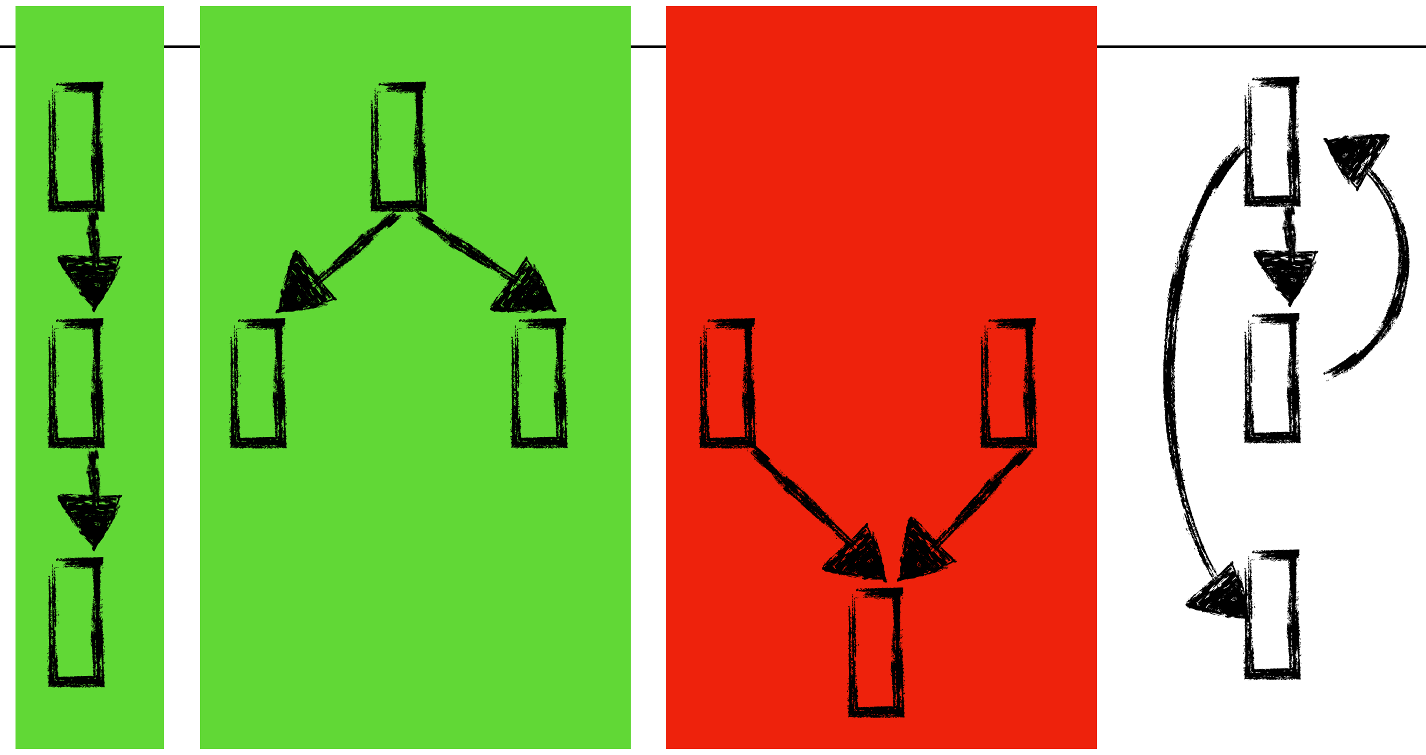

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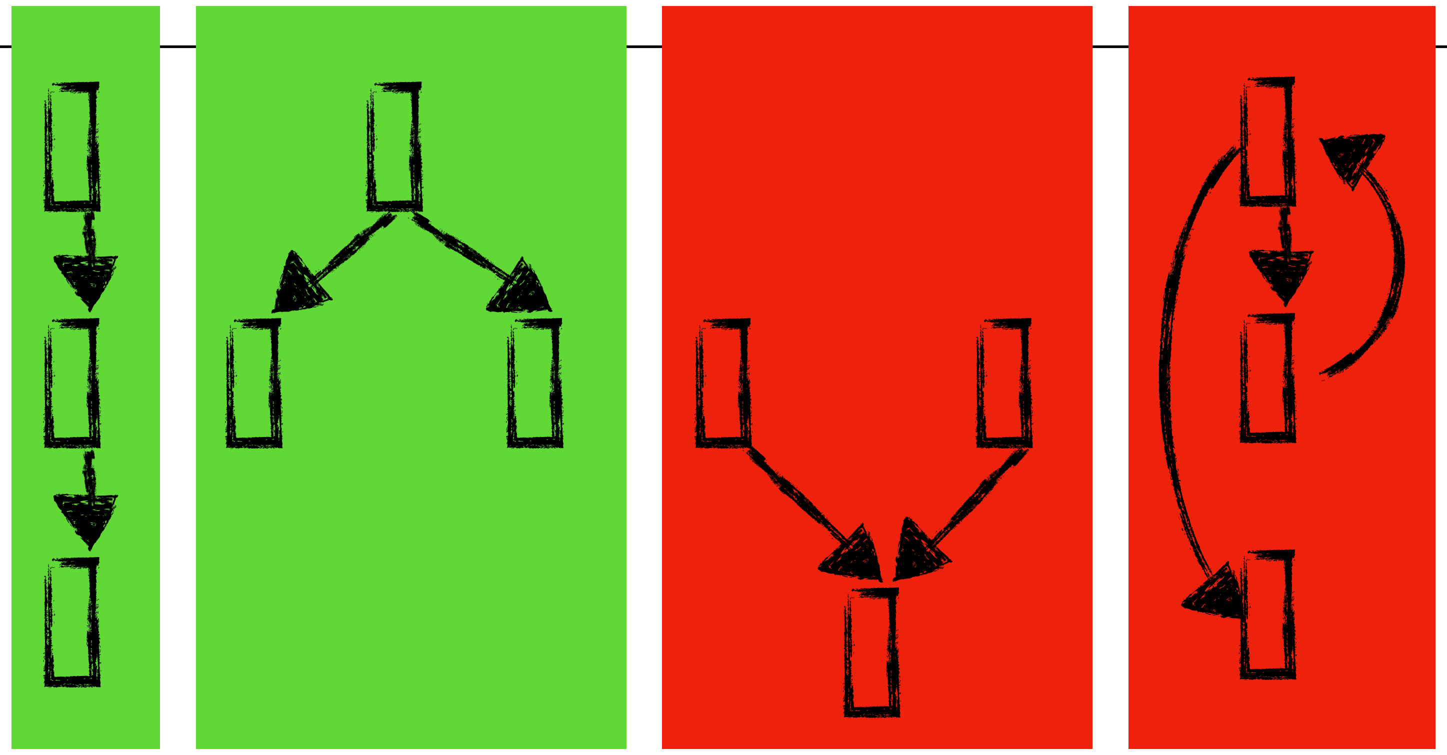

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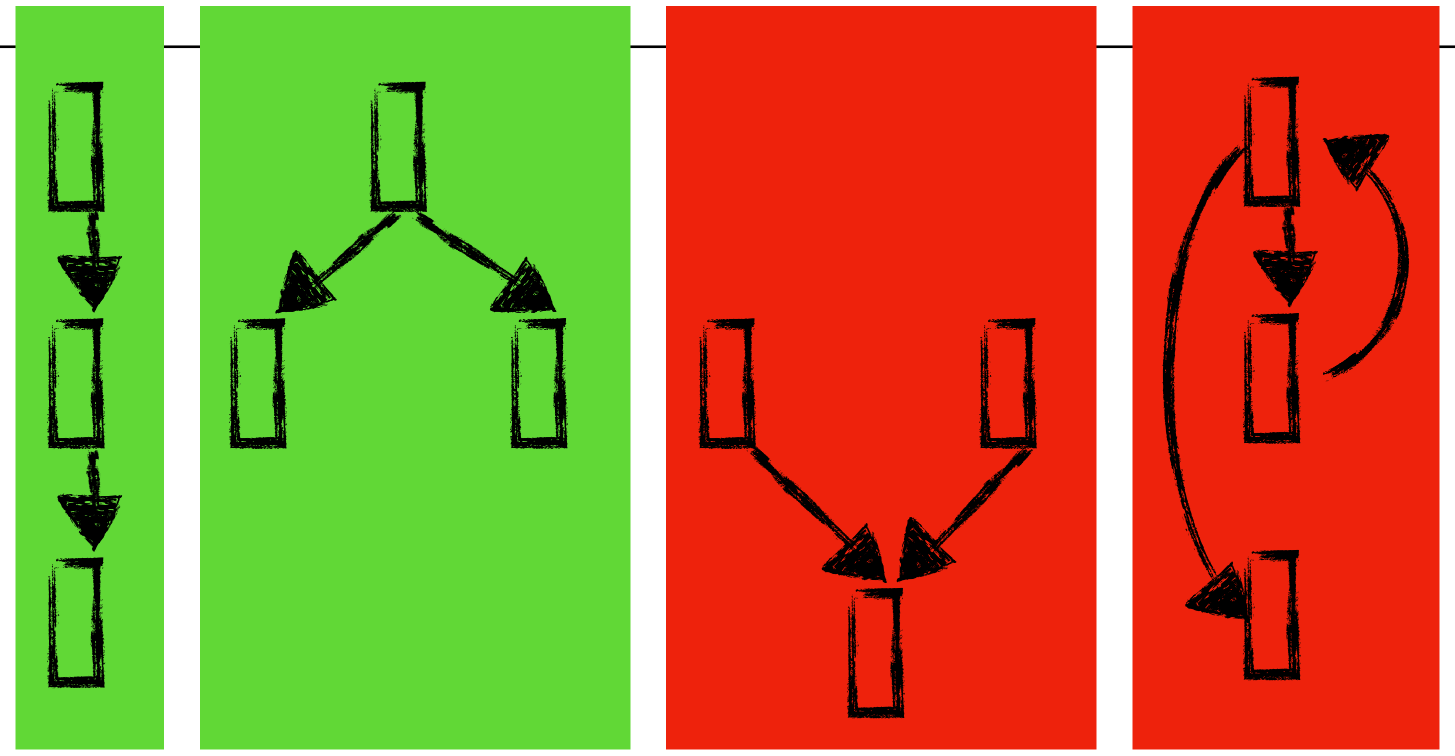

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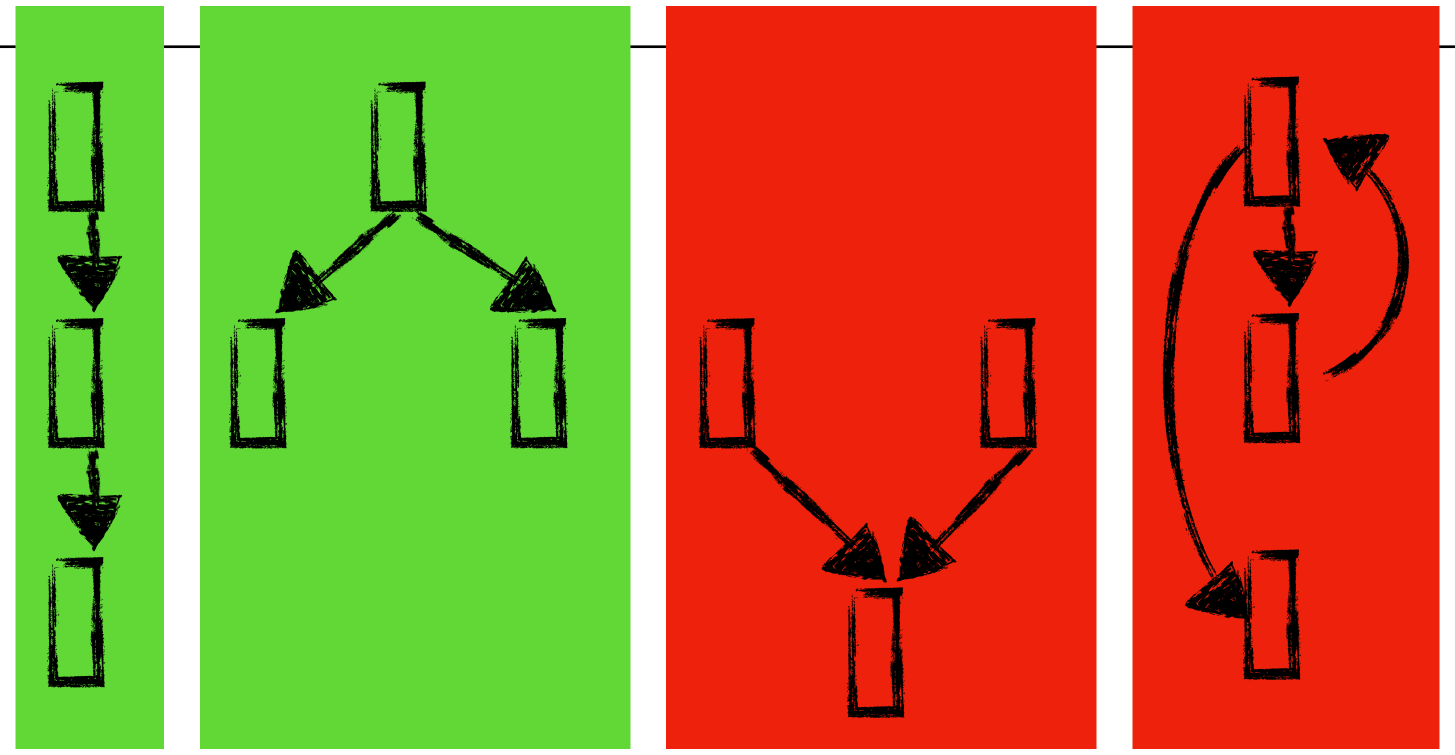

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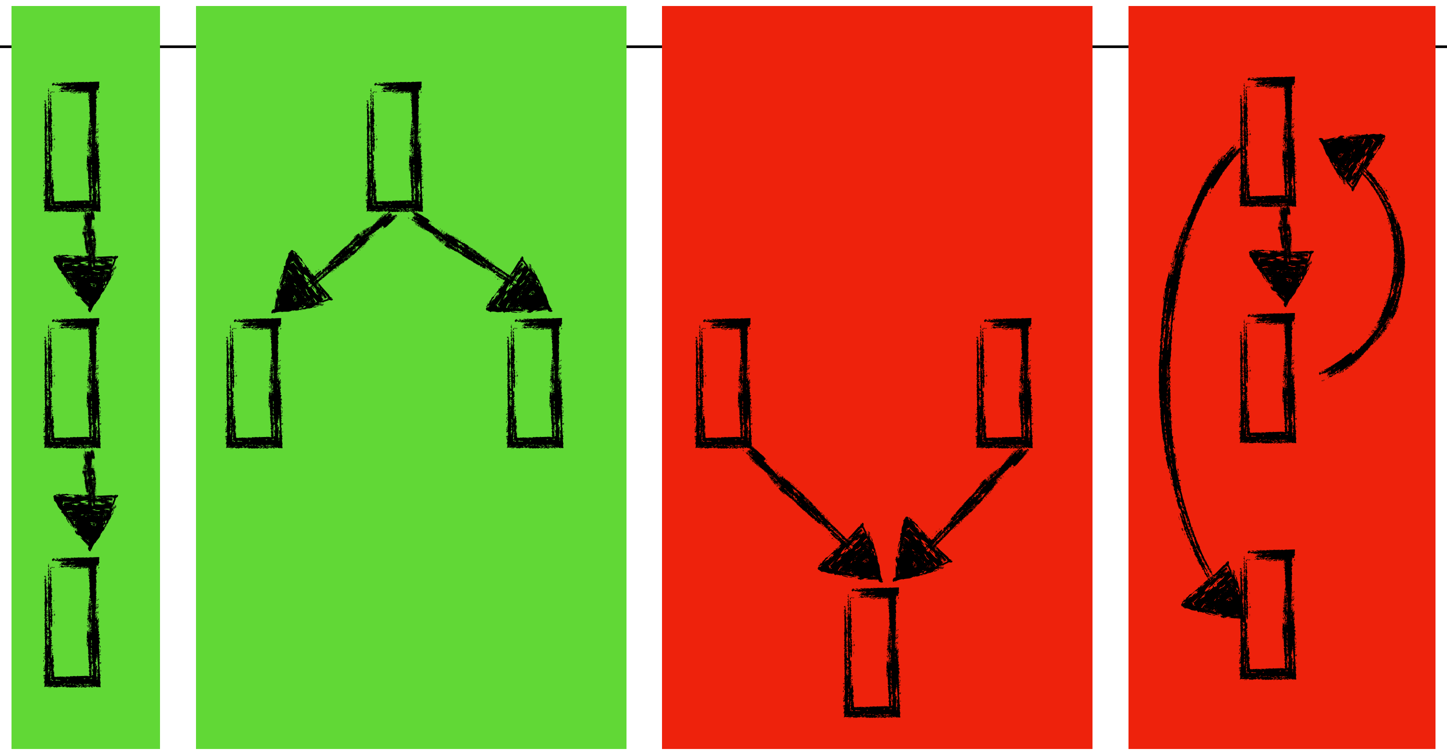

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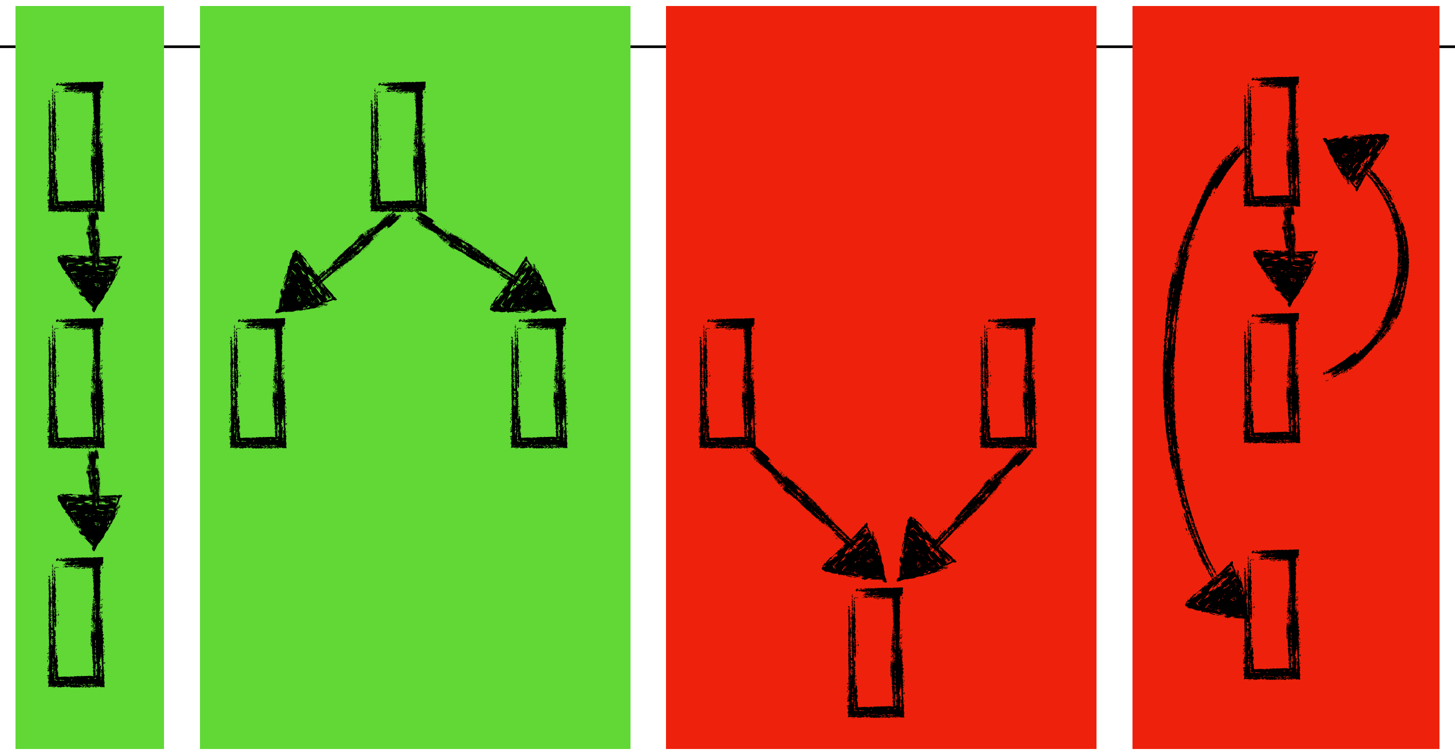

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Using Lattices

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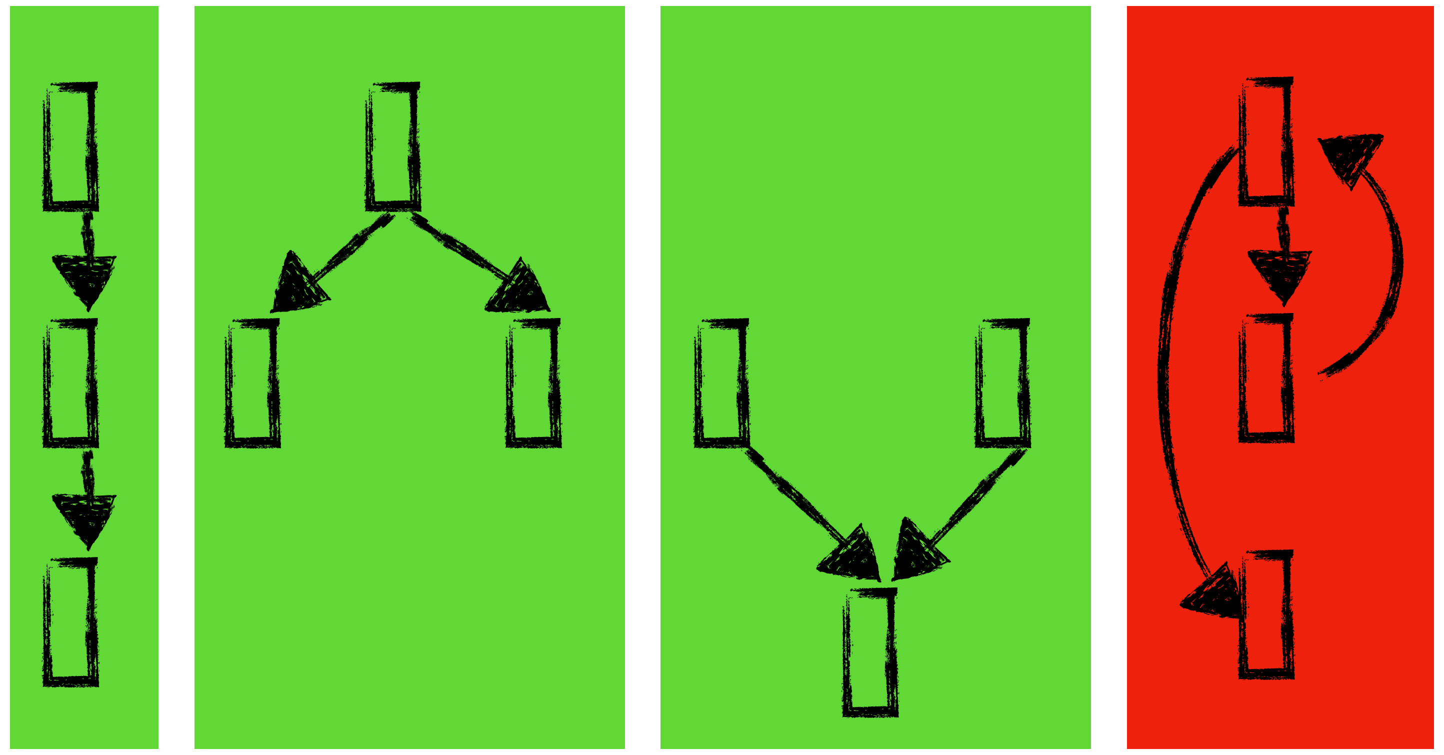

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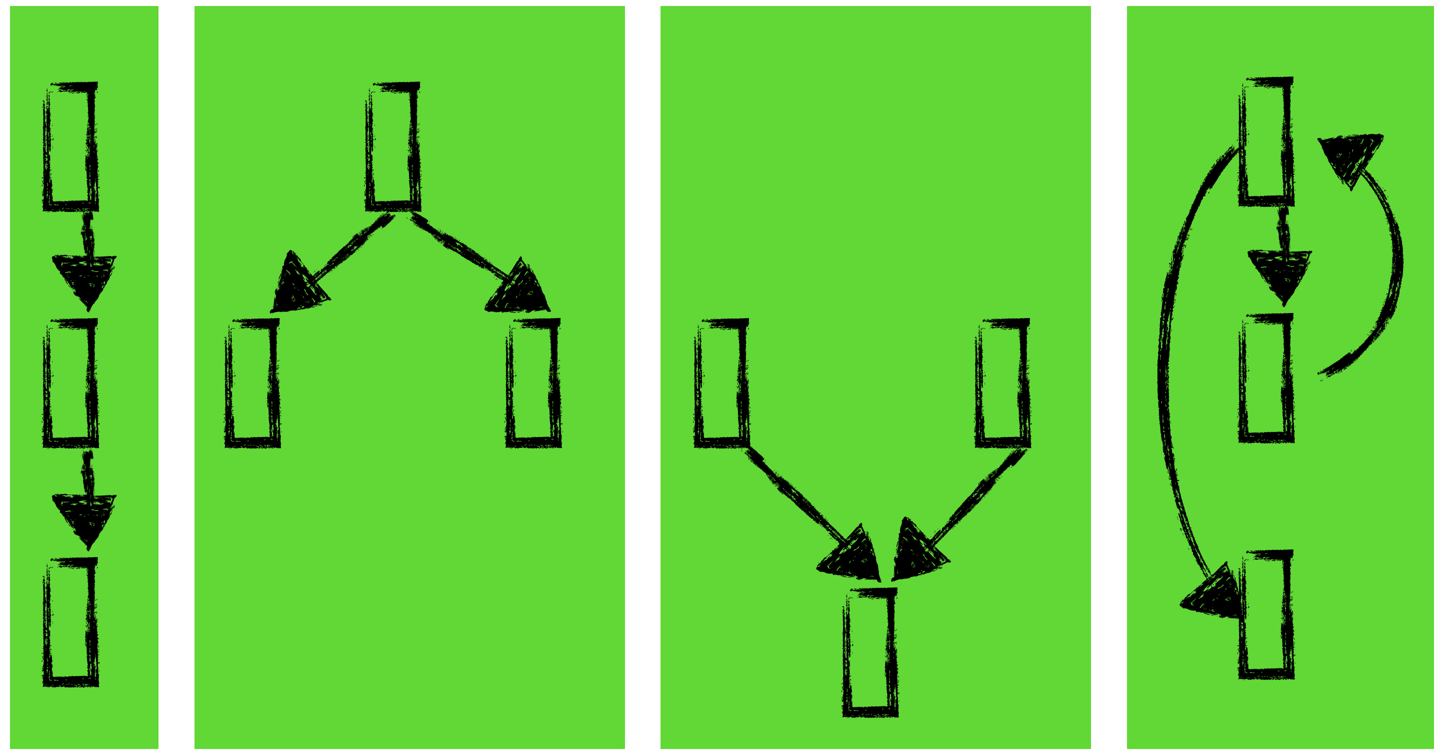

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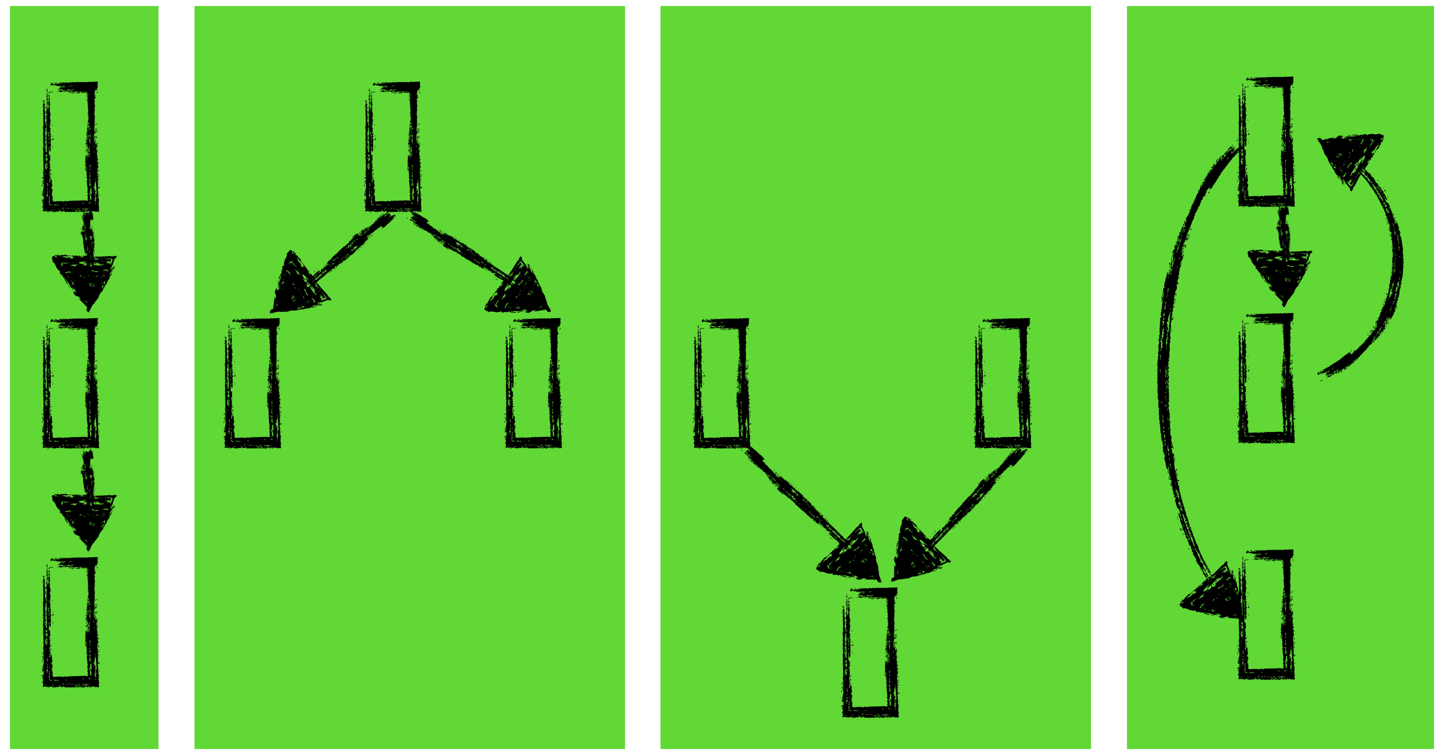

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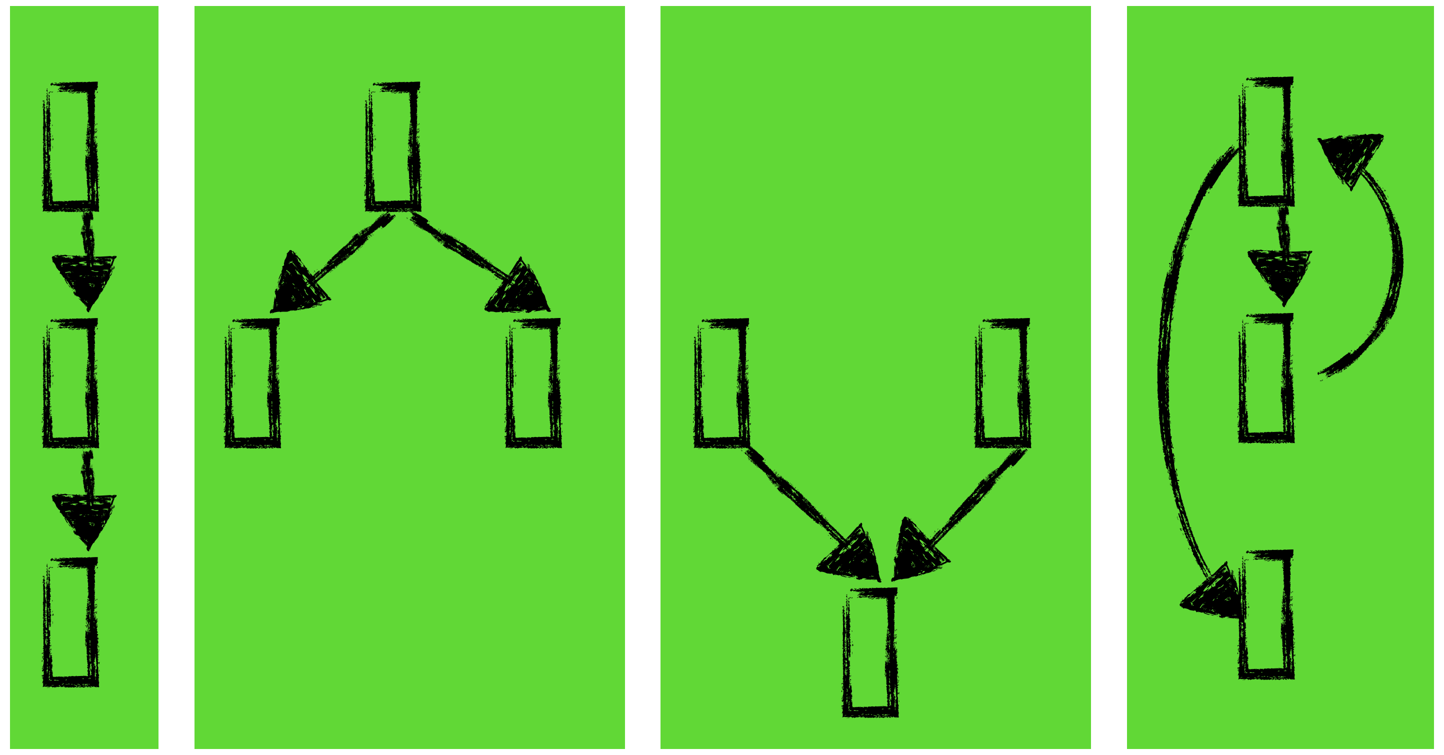

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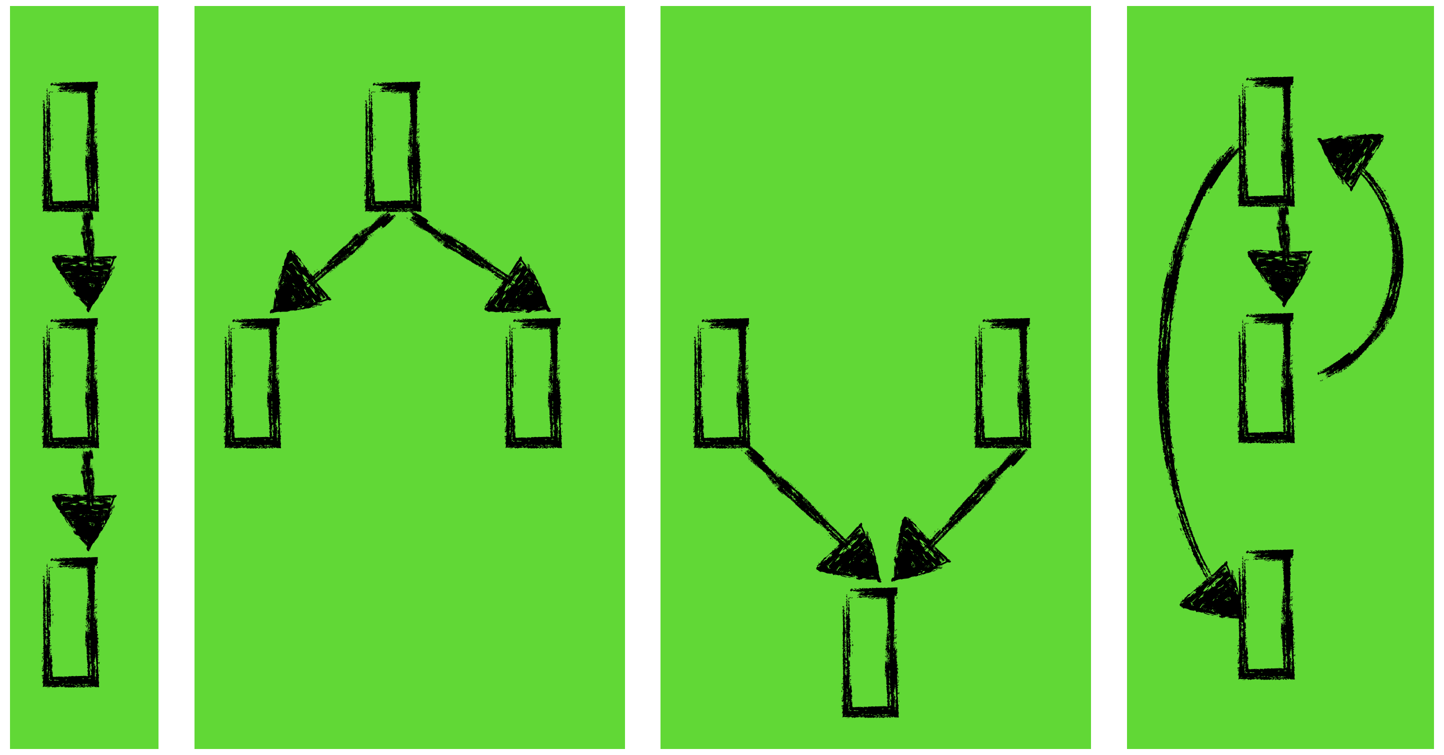

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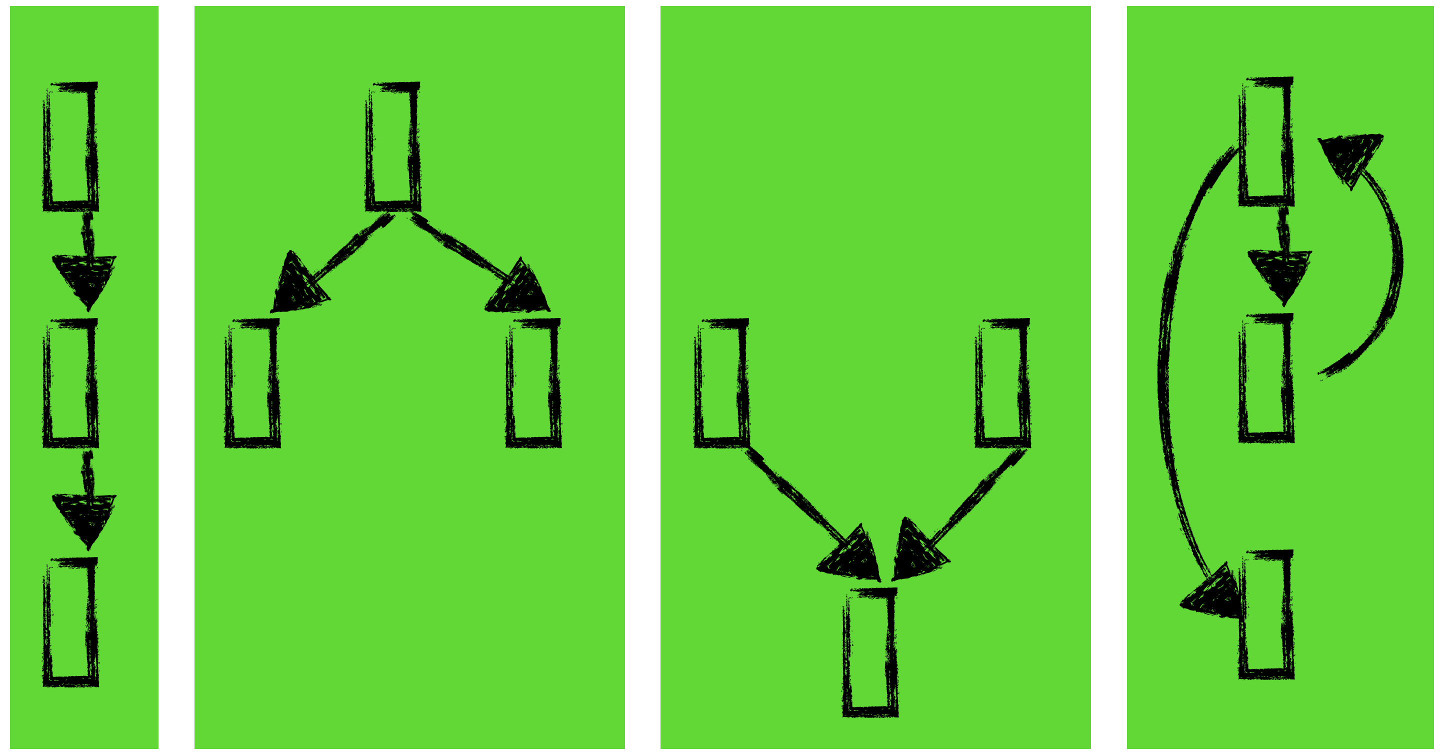

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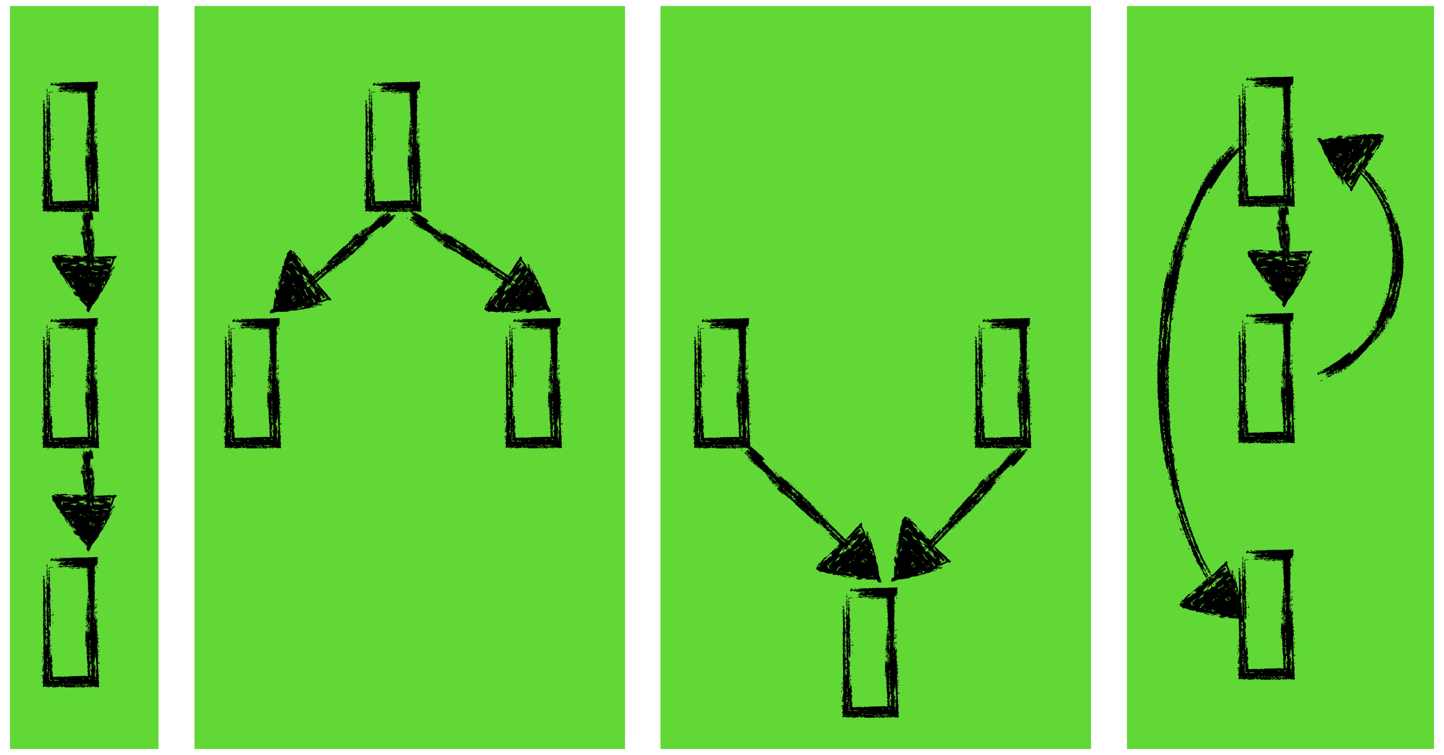

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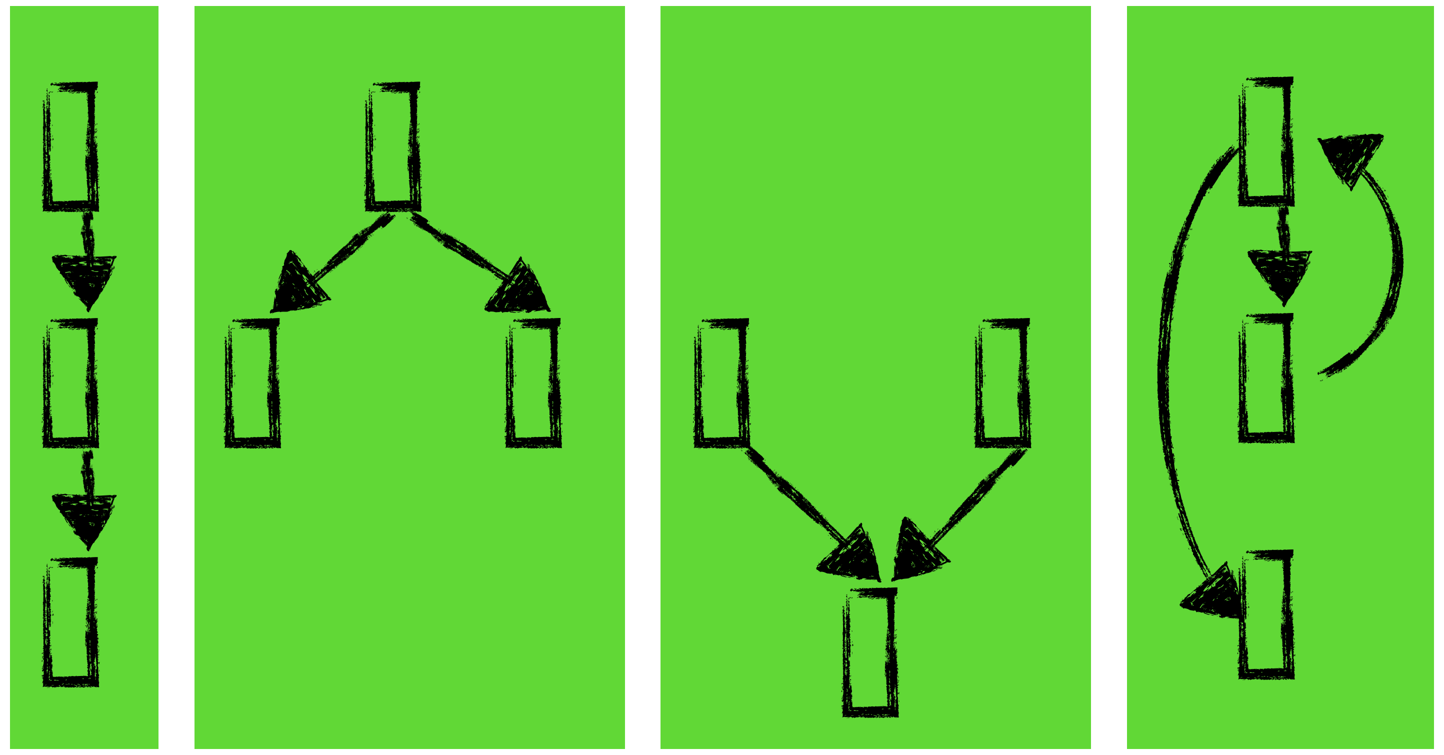

If **initial_value == bottom** and a transfer function is identity: traversal will stop there, so don't just start from the **start_node**

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FlowSpec Design

Framework Overview

Control-flow graph

- graph
- start node
- reverse beforehand if backward analysis

Lattice instance for data-flow information:

- Lattice L
- Least Upper Bound $\sqcup$
- Bottom value $\bot \in L$

Initial data-flow information for start node

Transfer function $f : (L \to L)$ per control-flow graph node

- Denotes the data-flow effect of the CFG node

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Strongly Connected Component (SCC) identification

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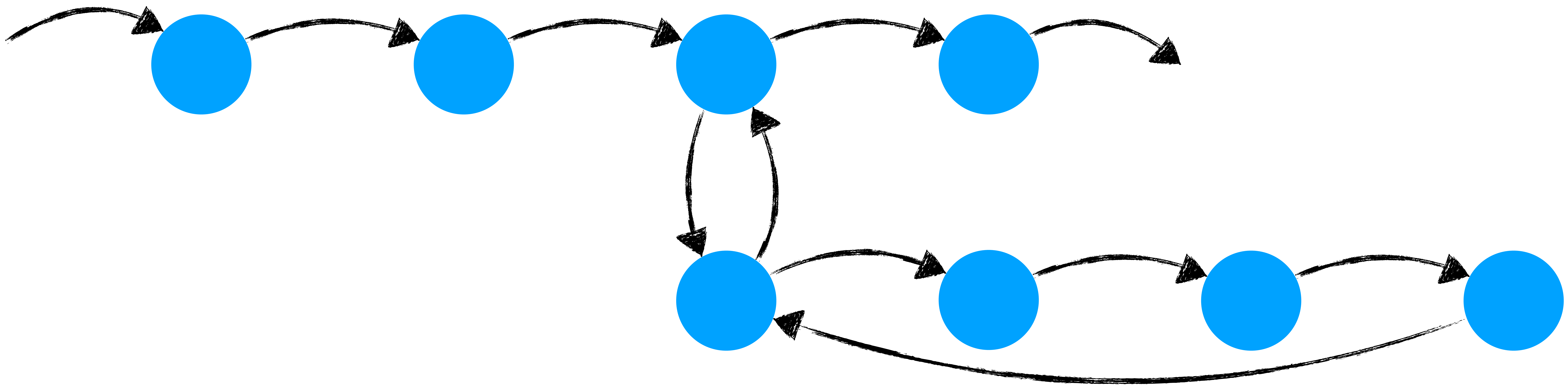

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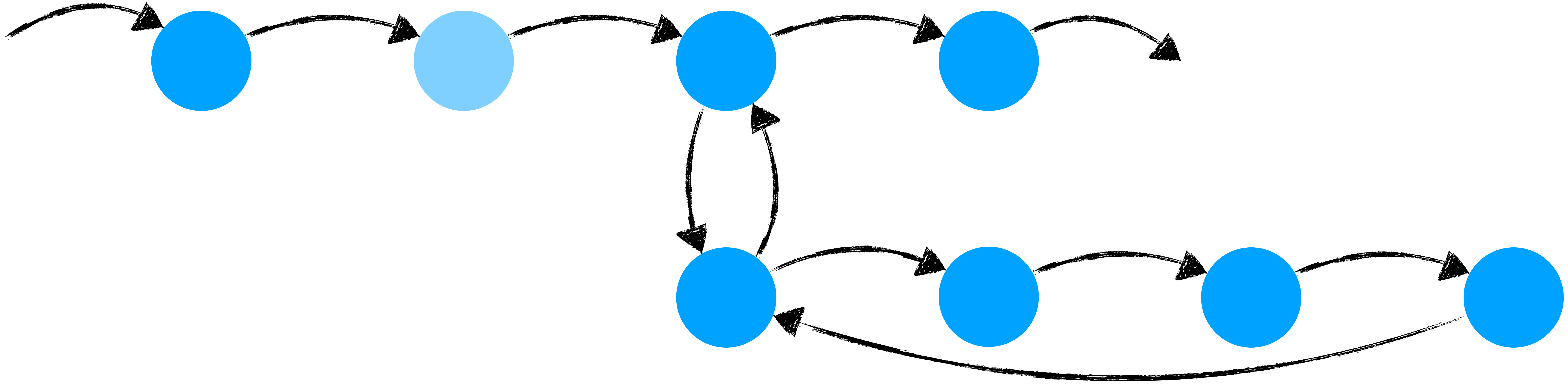

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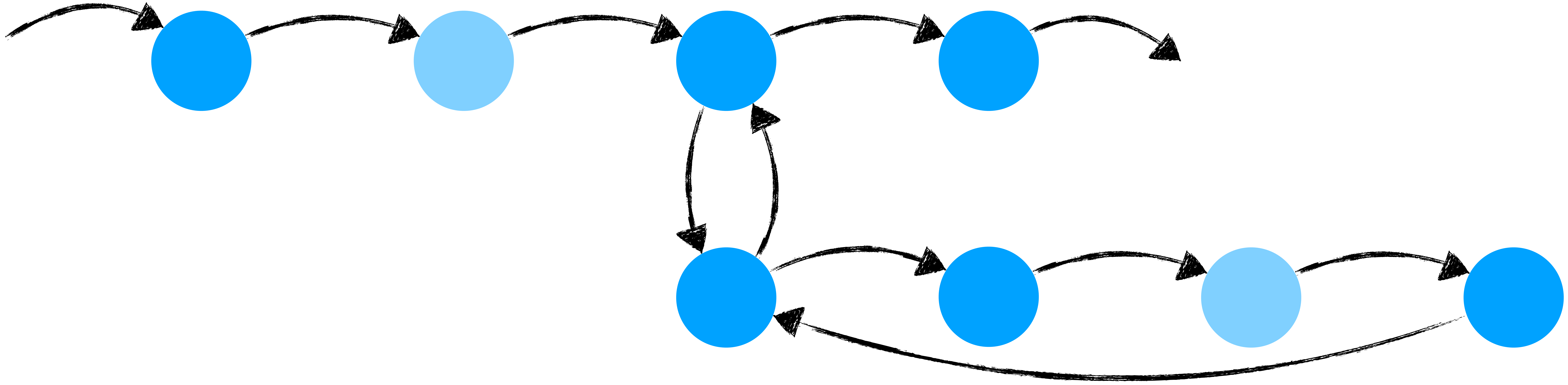

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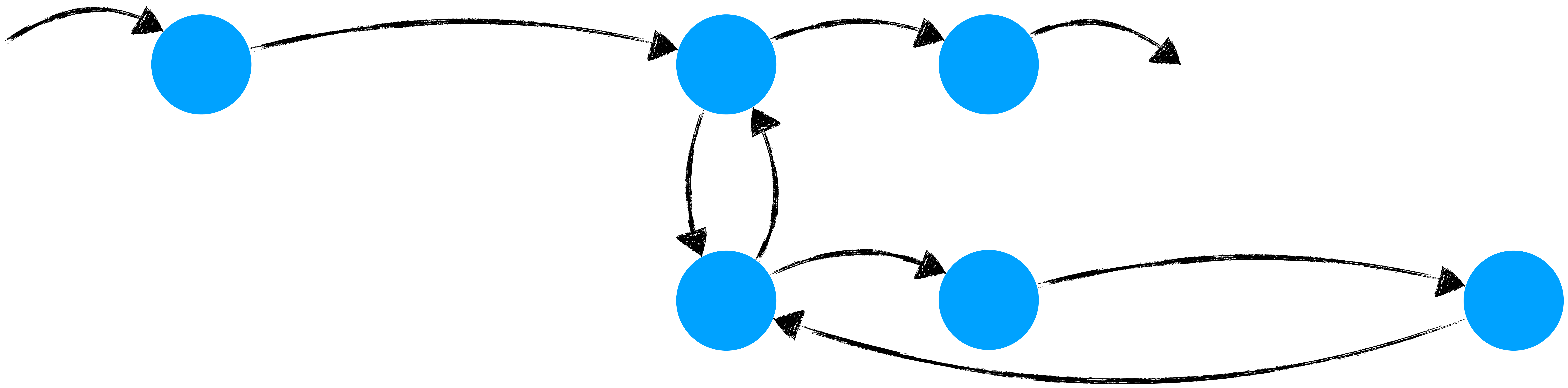

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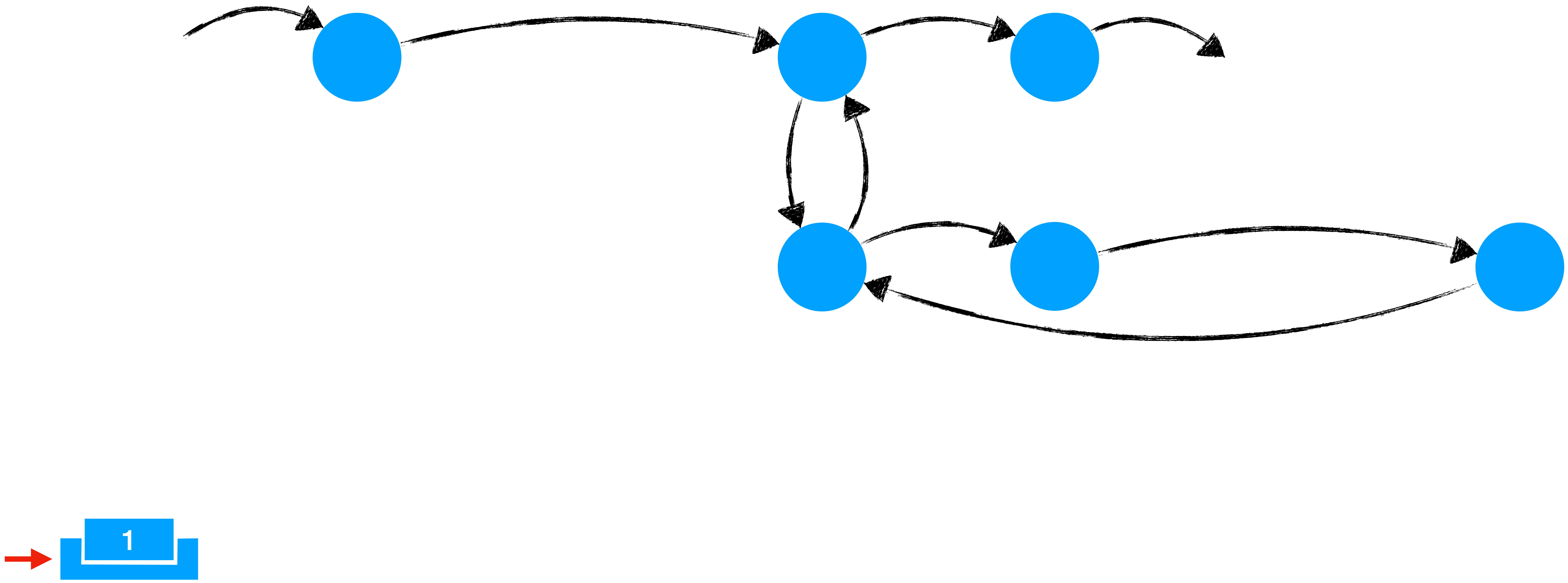

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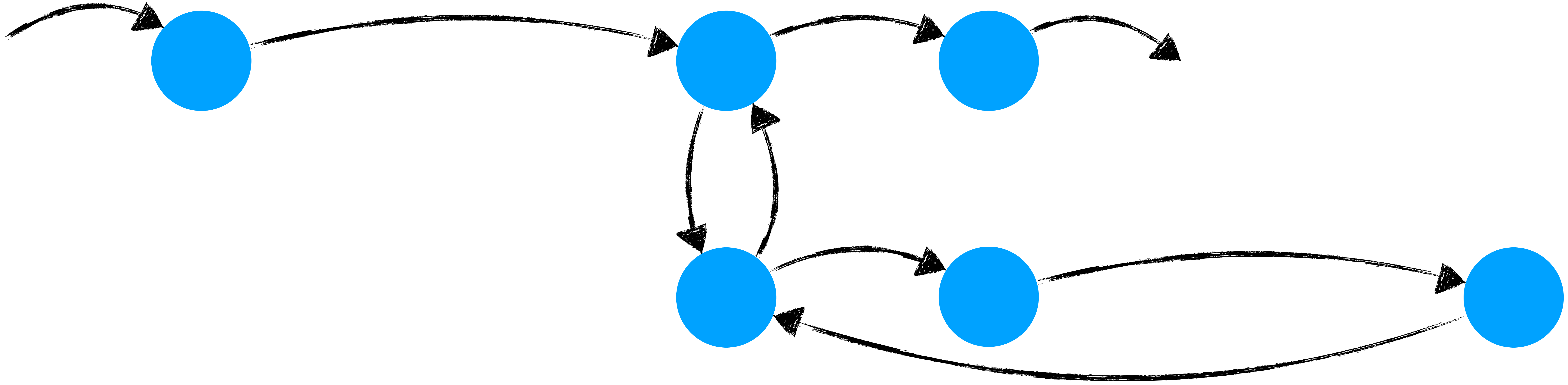

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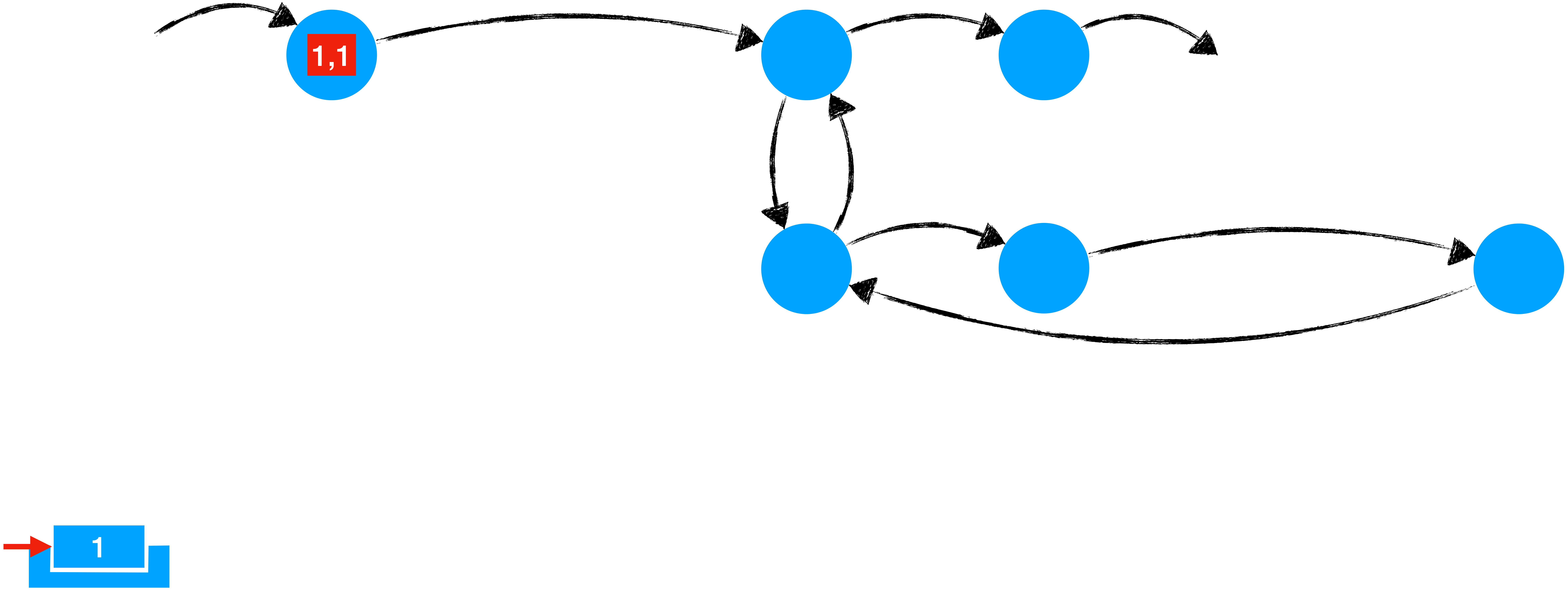

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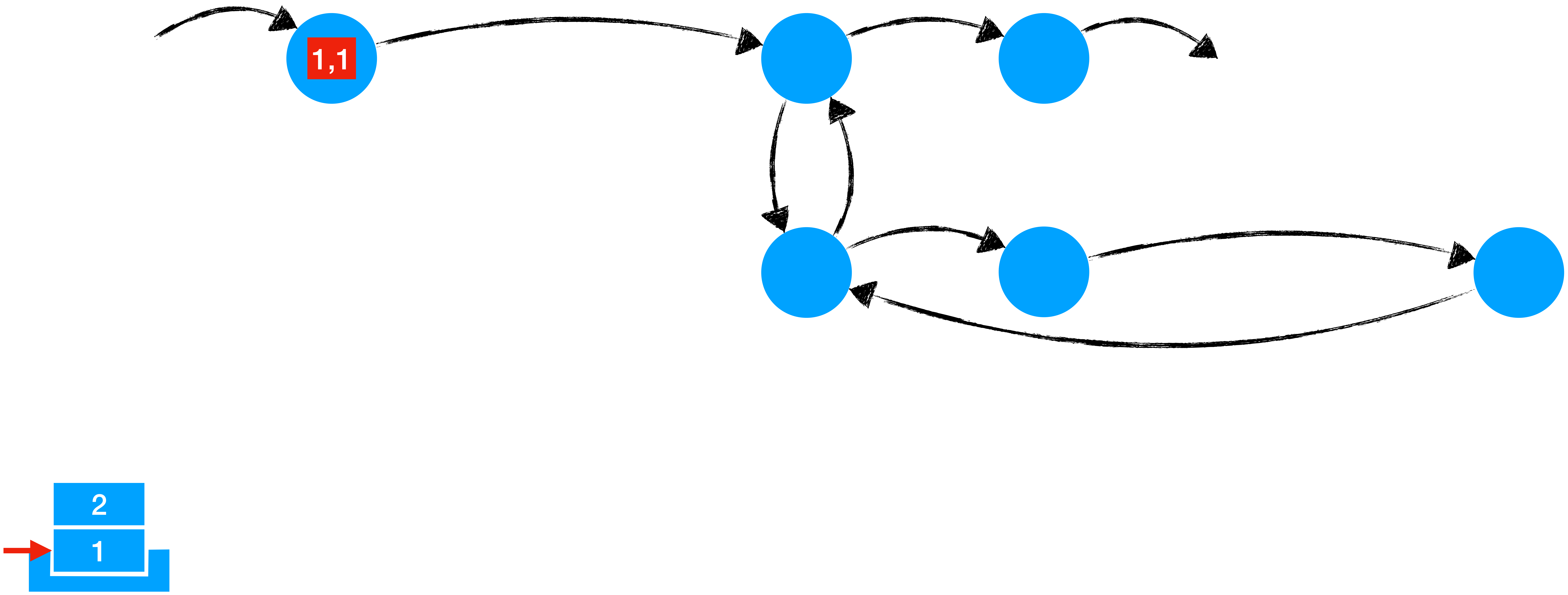

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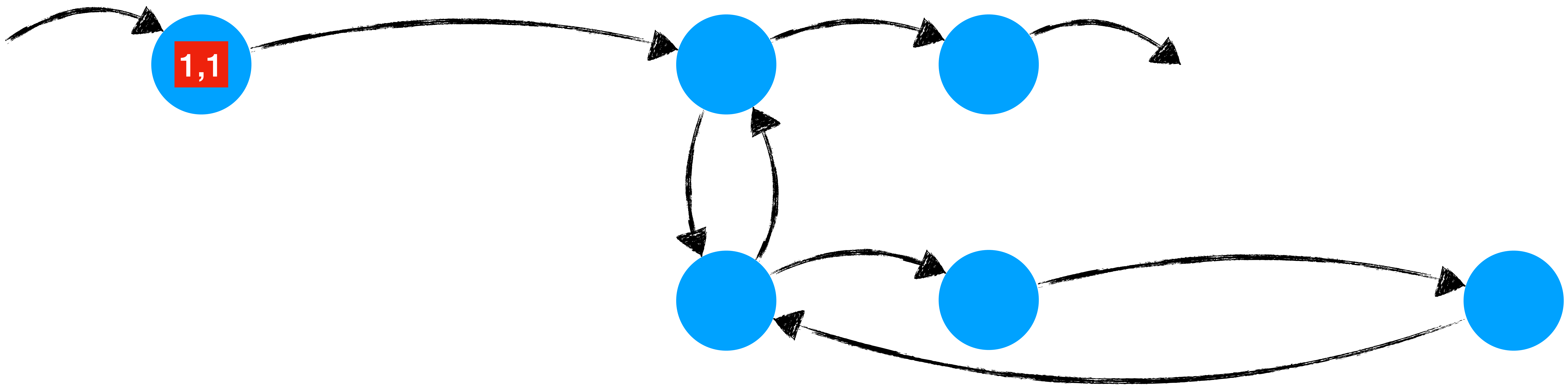

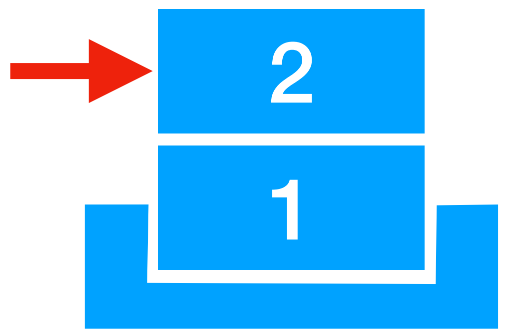

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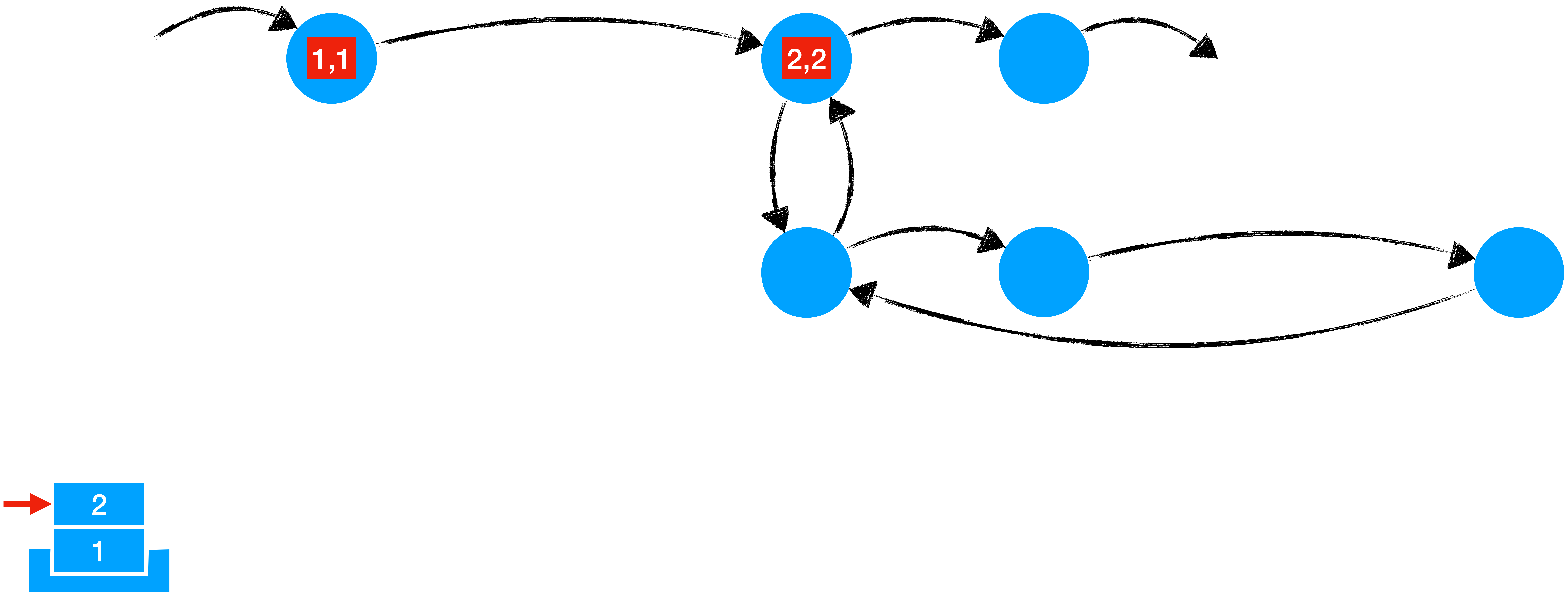

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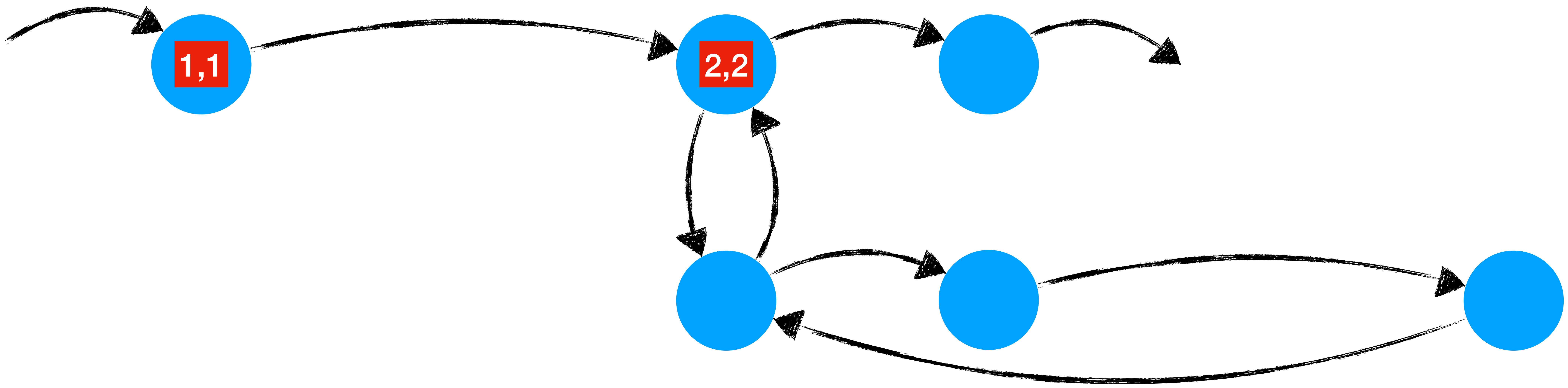

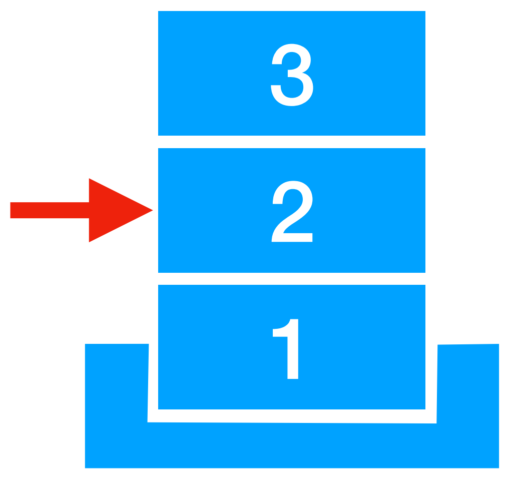

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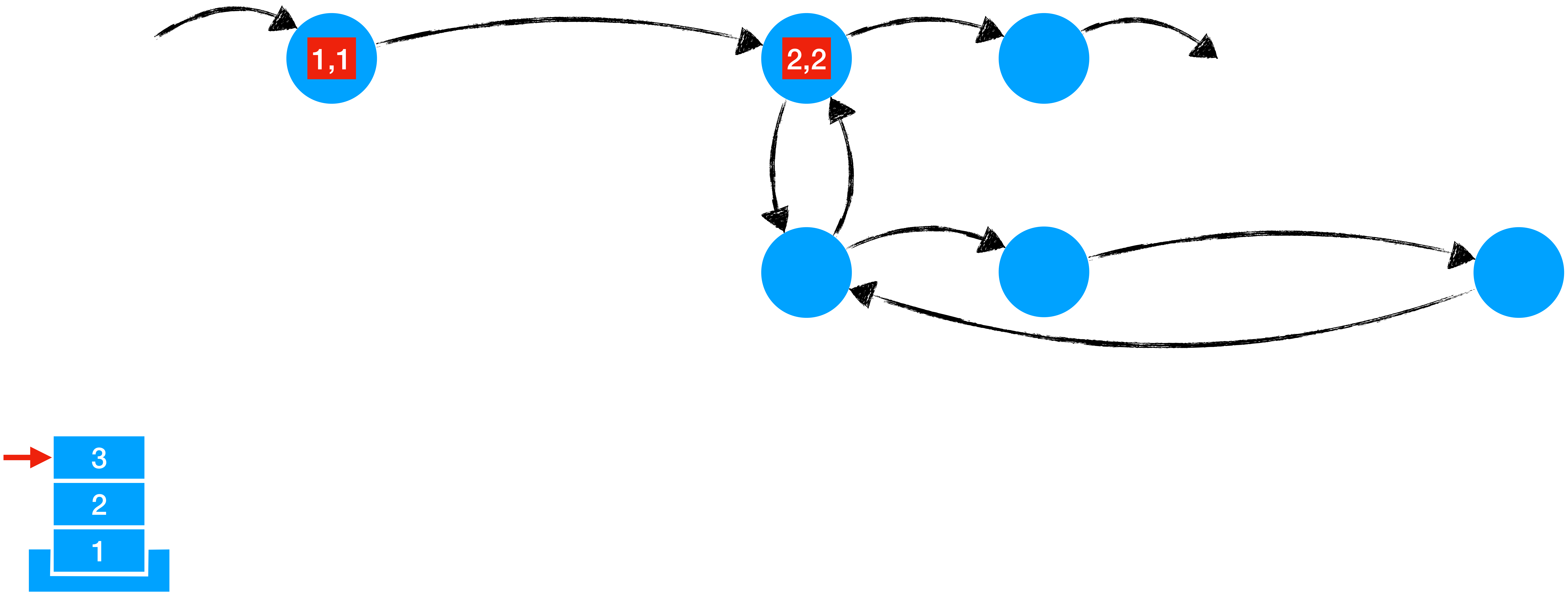

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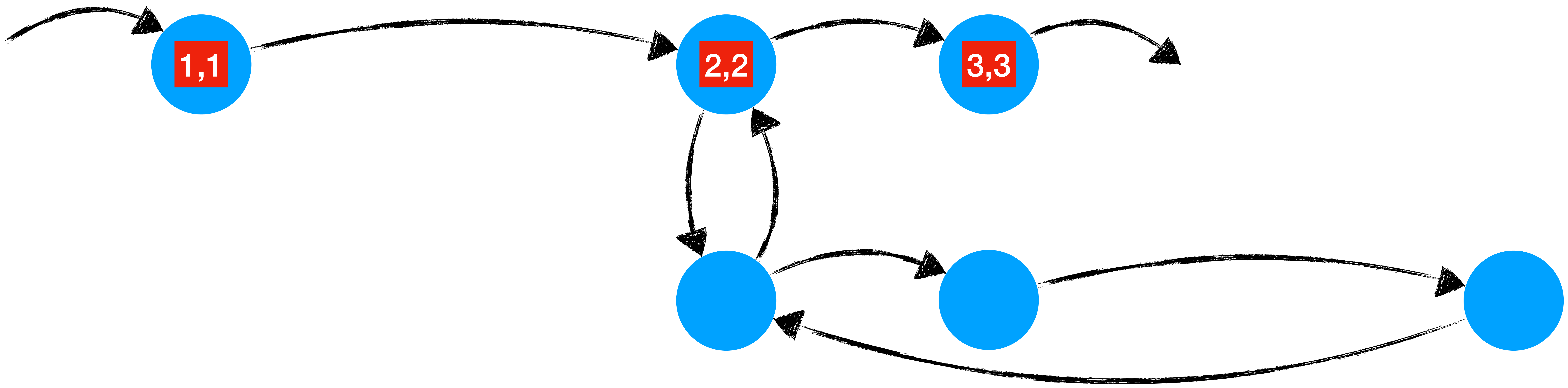

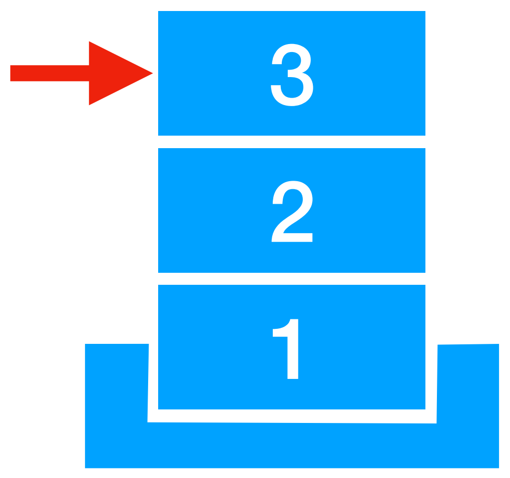

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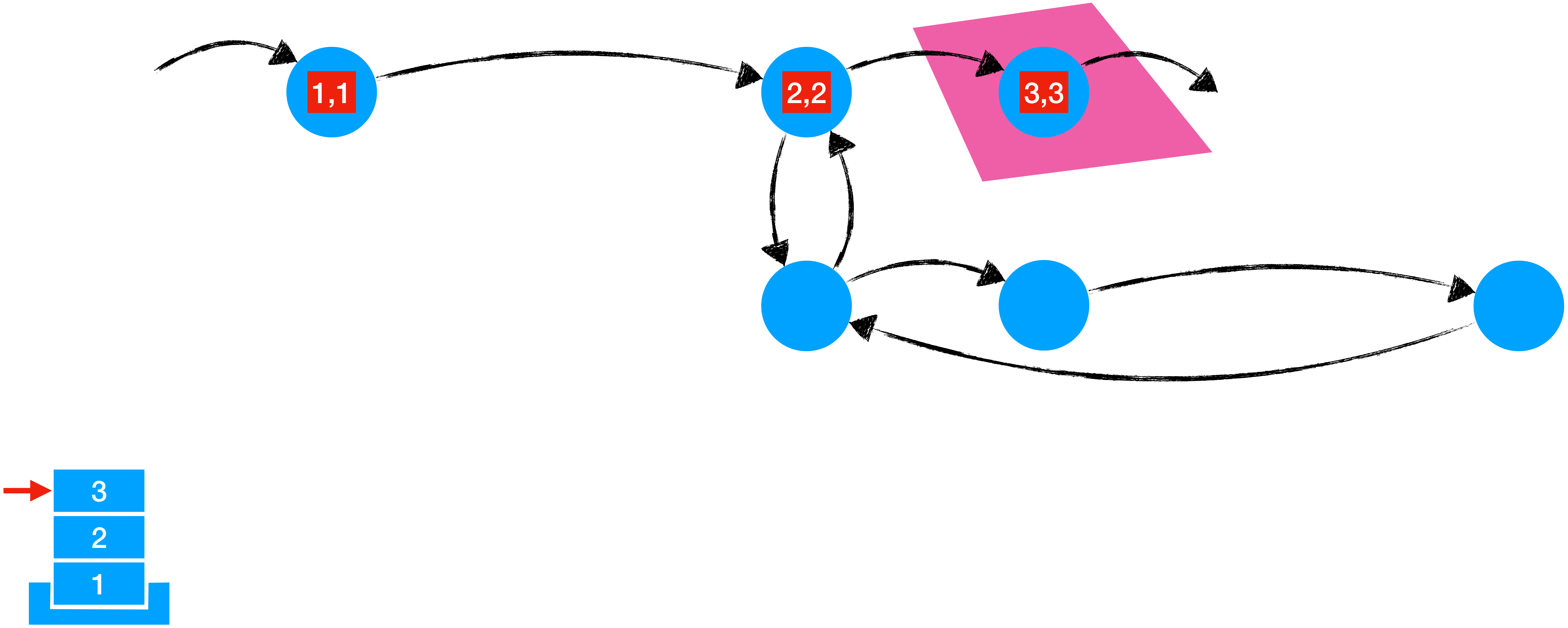

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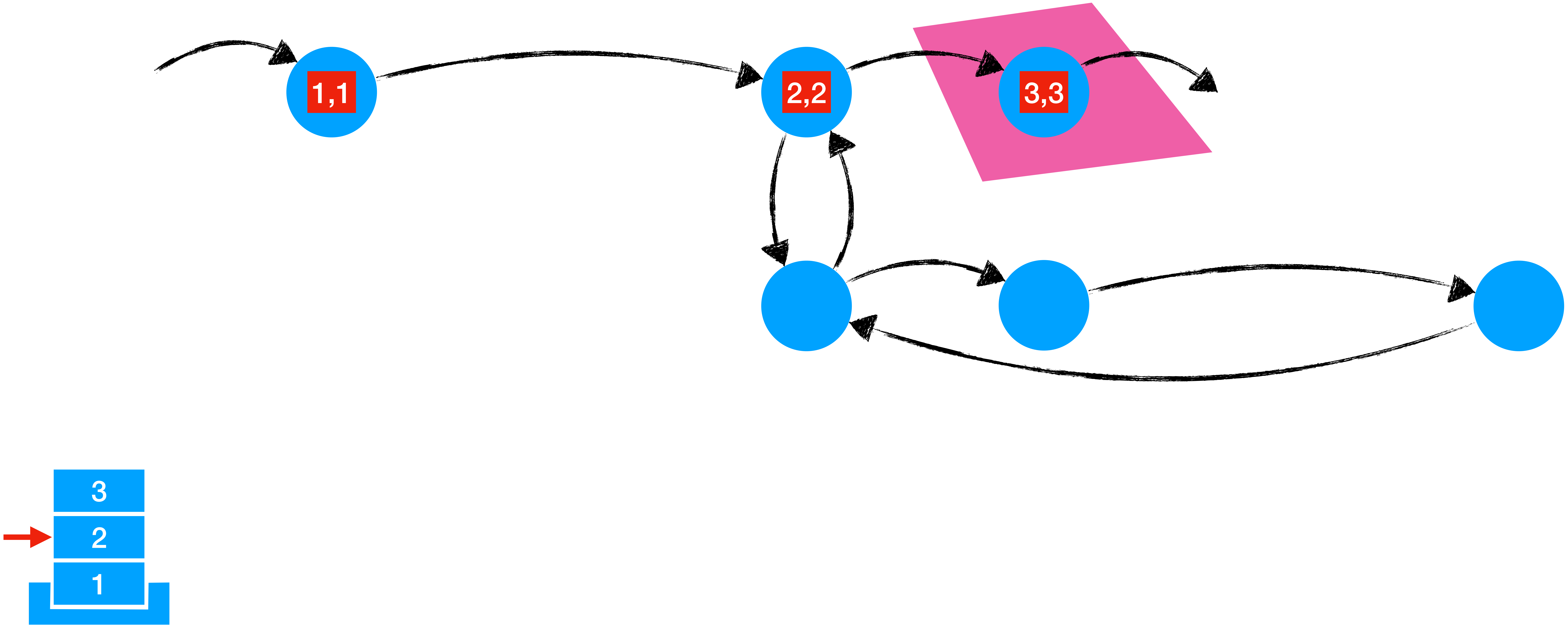

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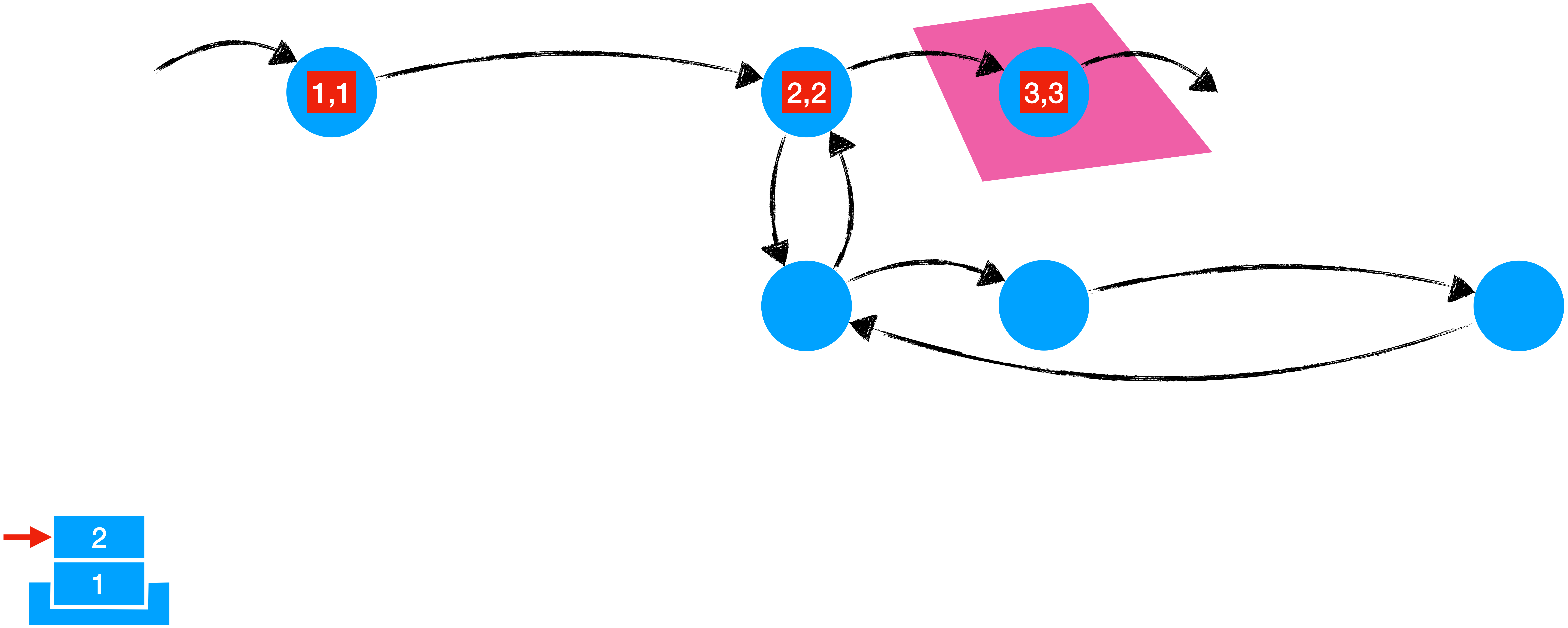

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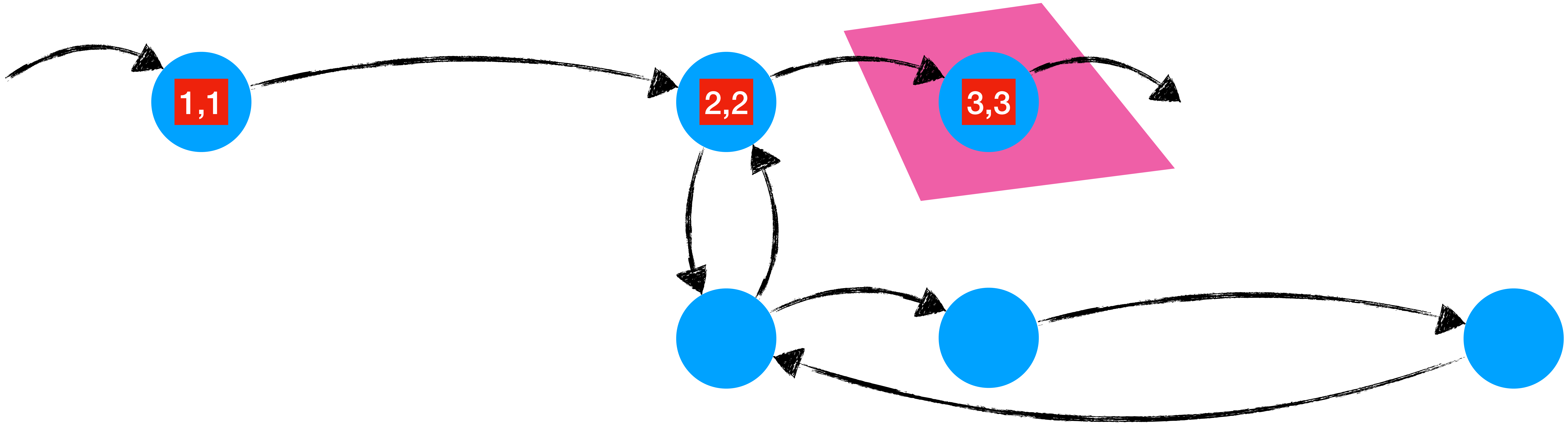

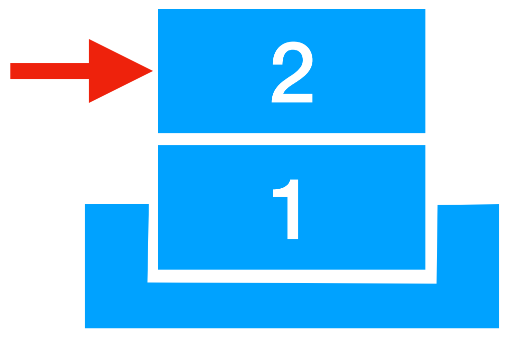

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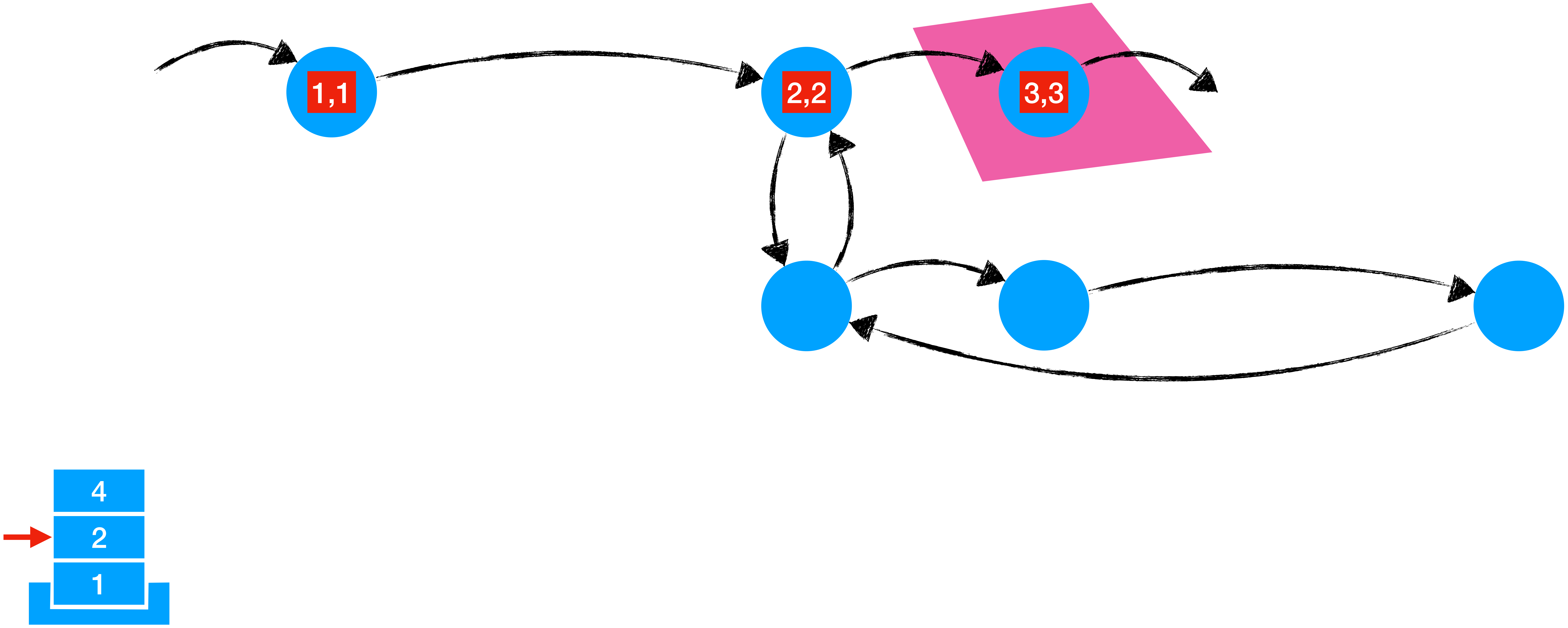

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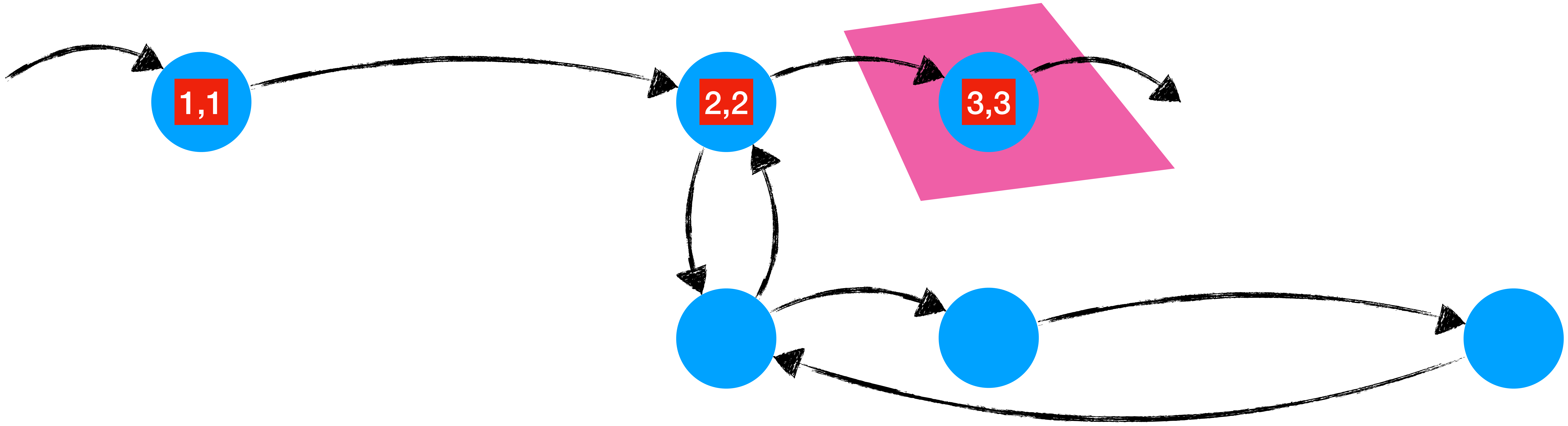

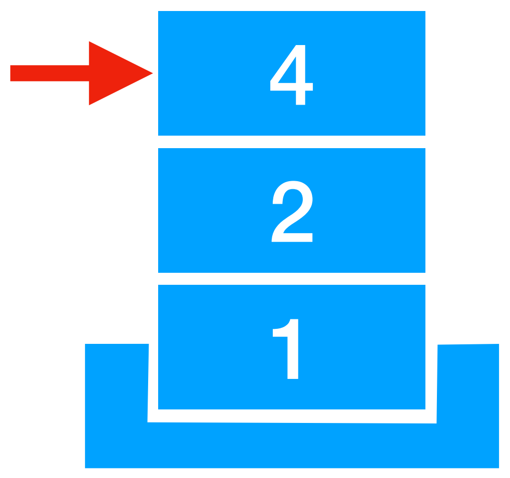

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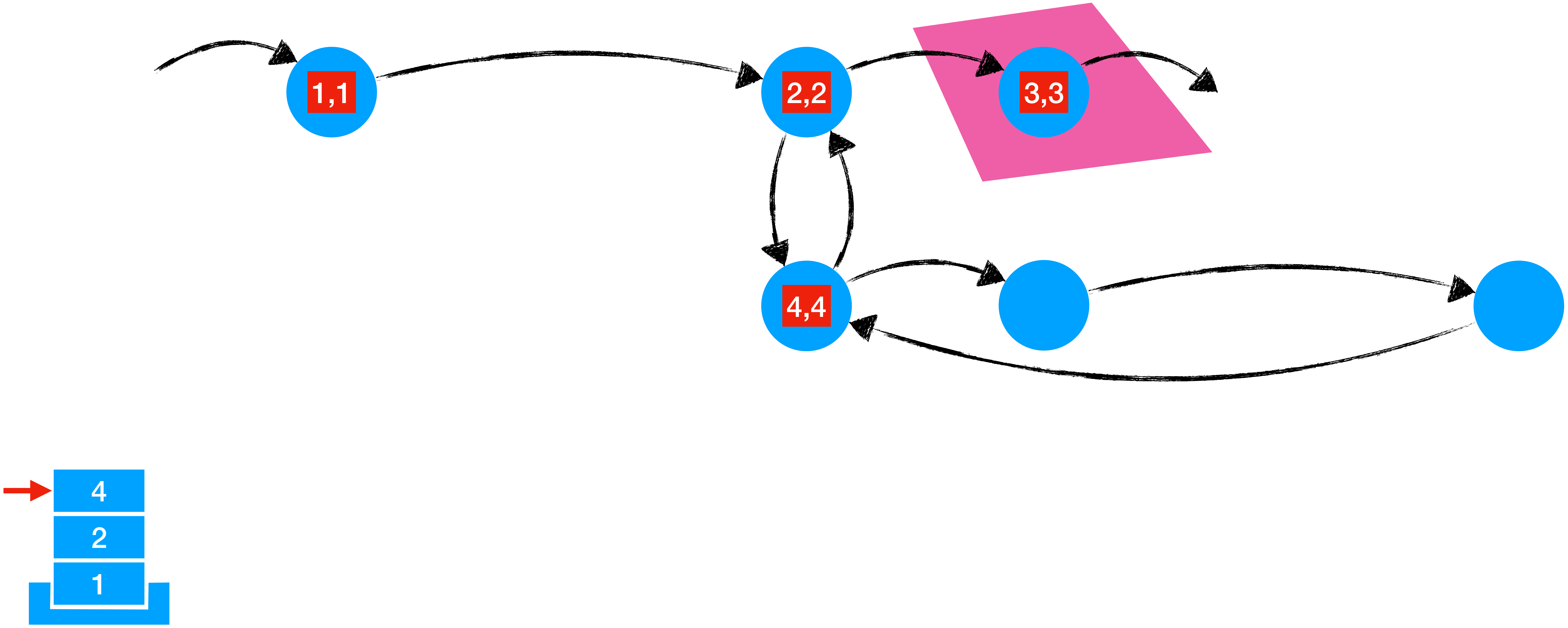

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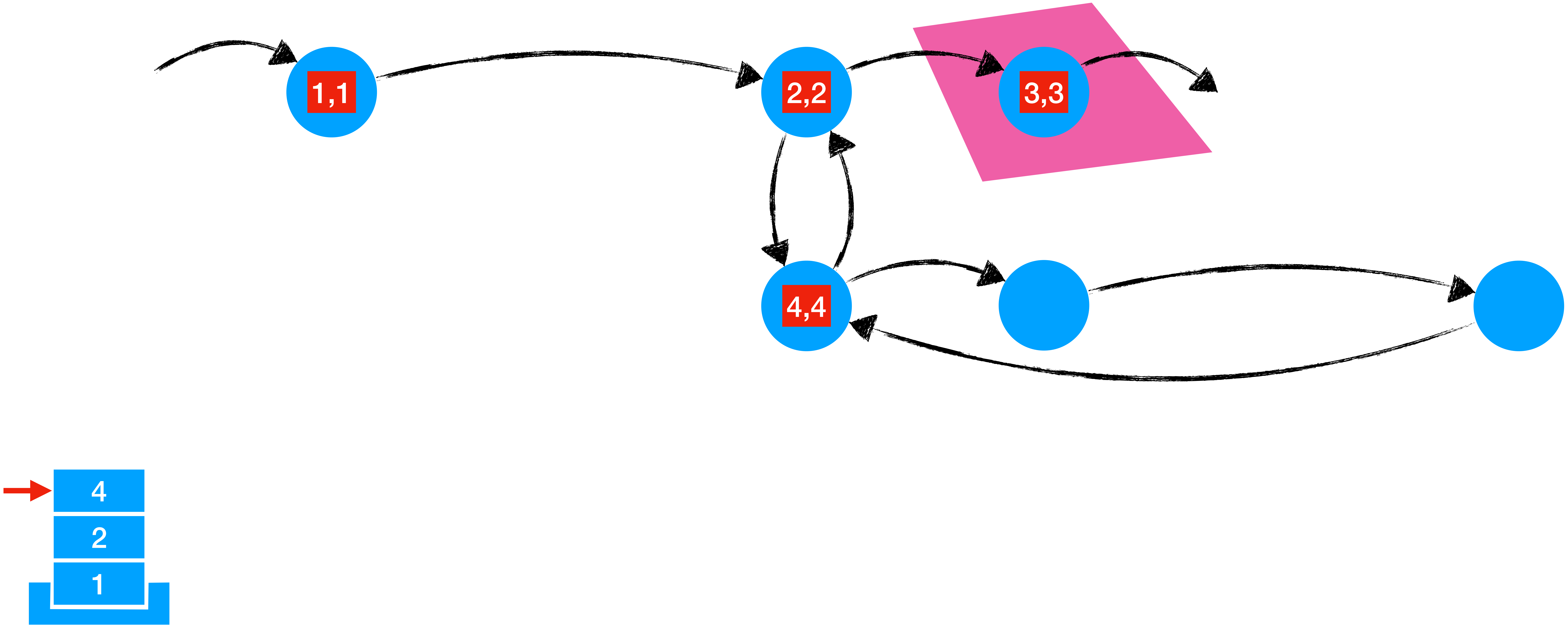

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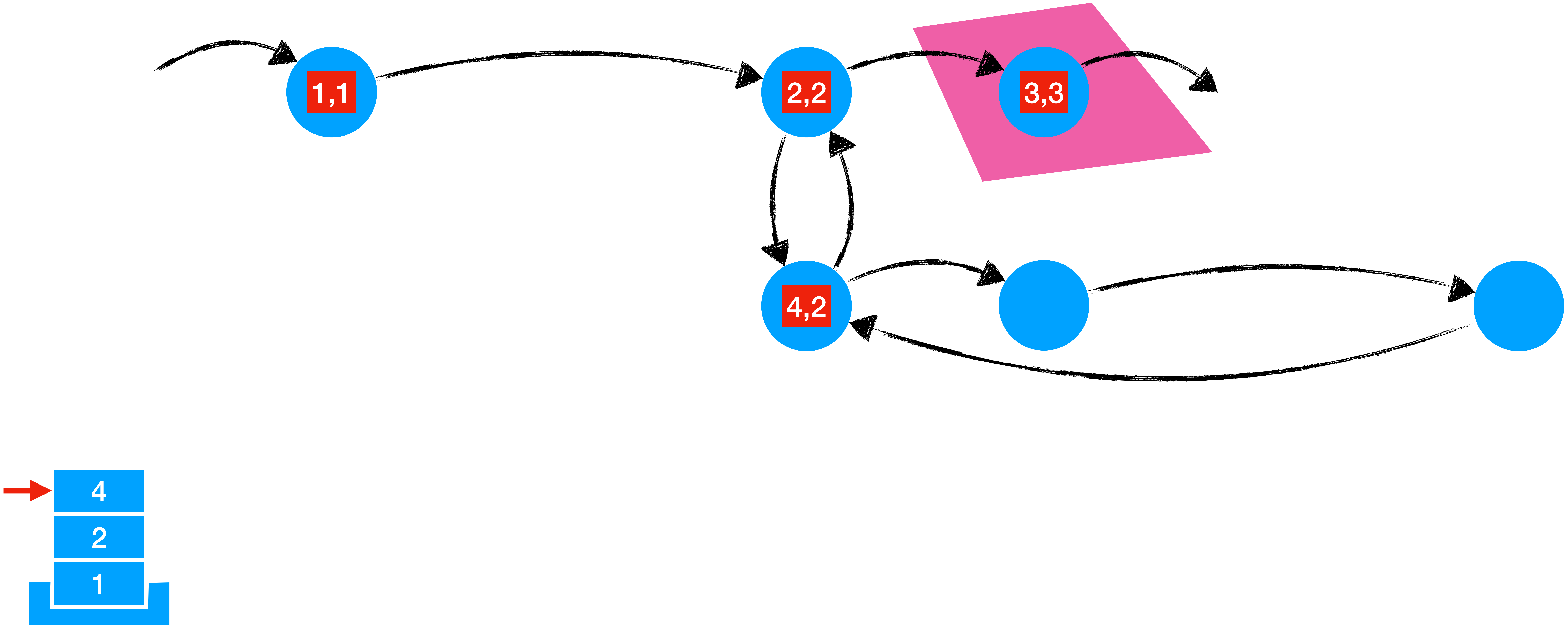

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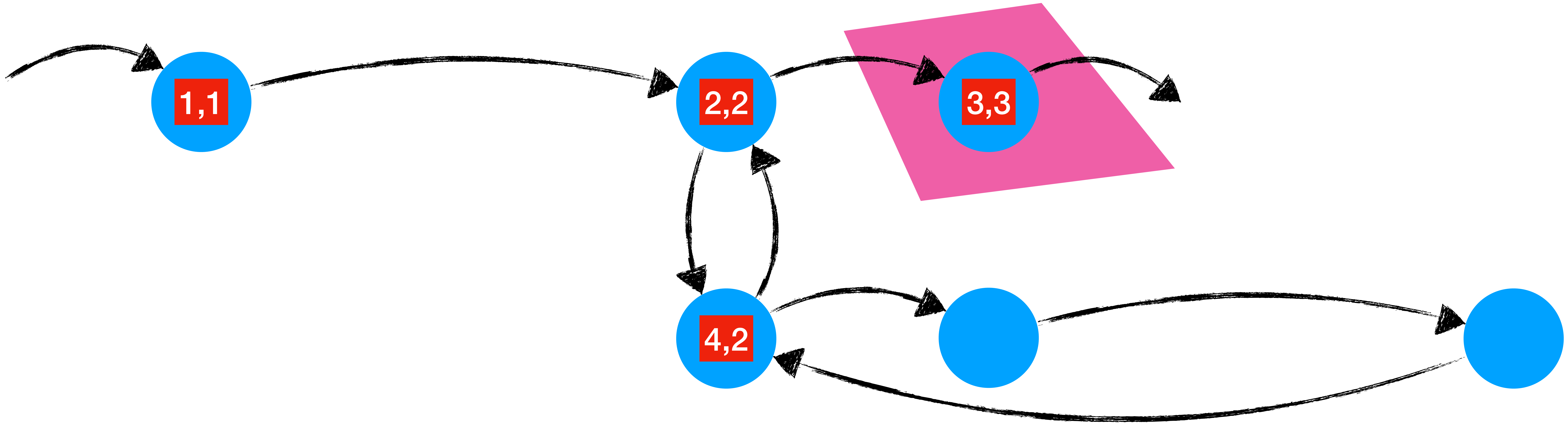

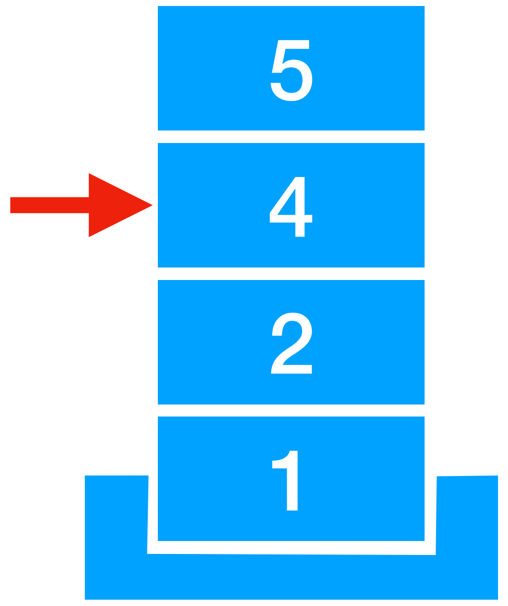

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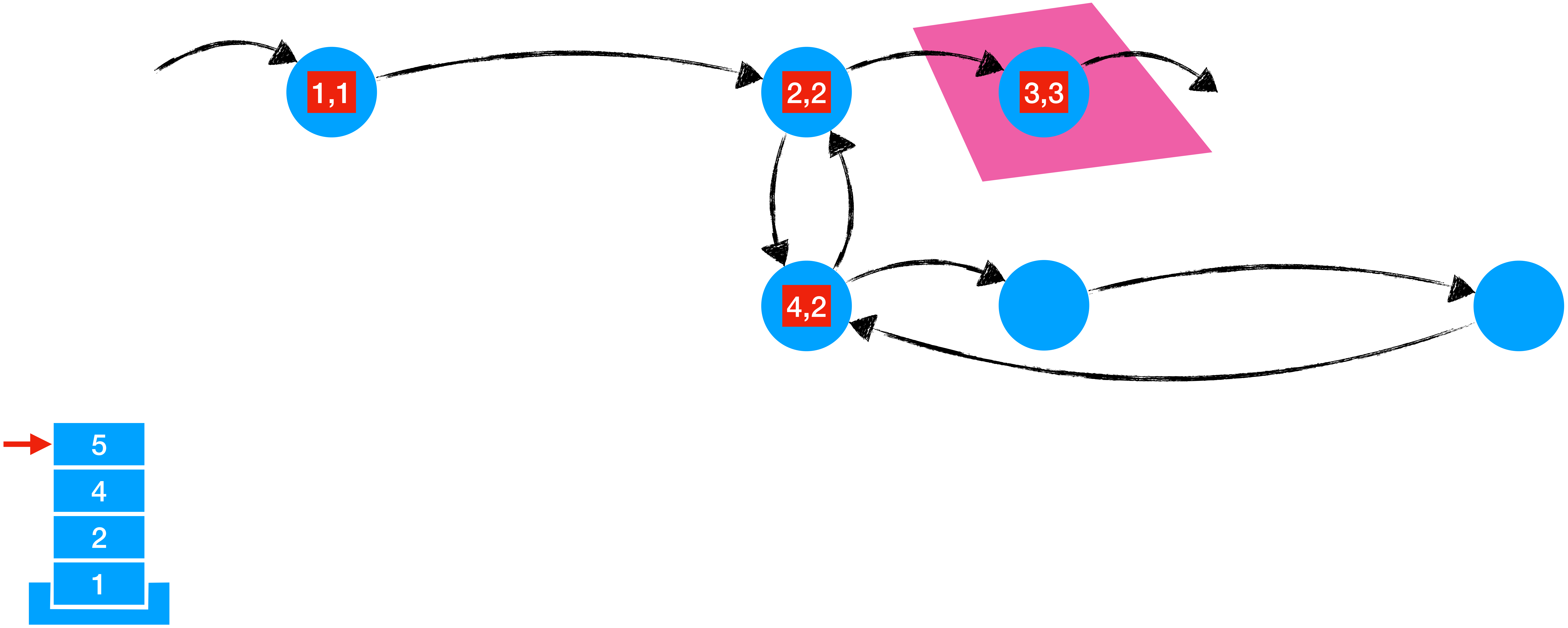

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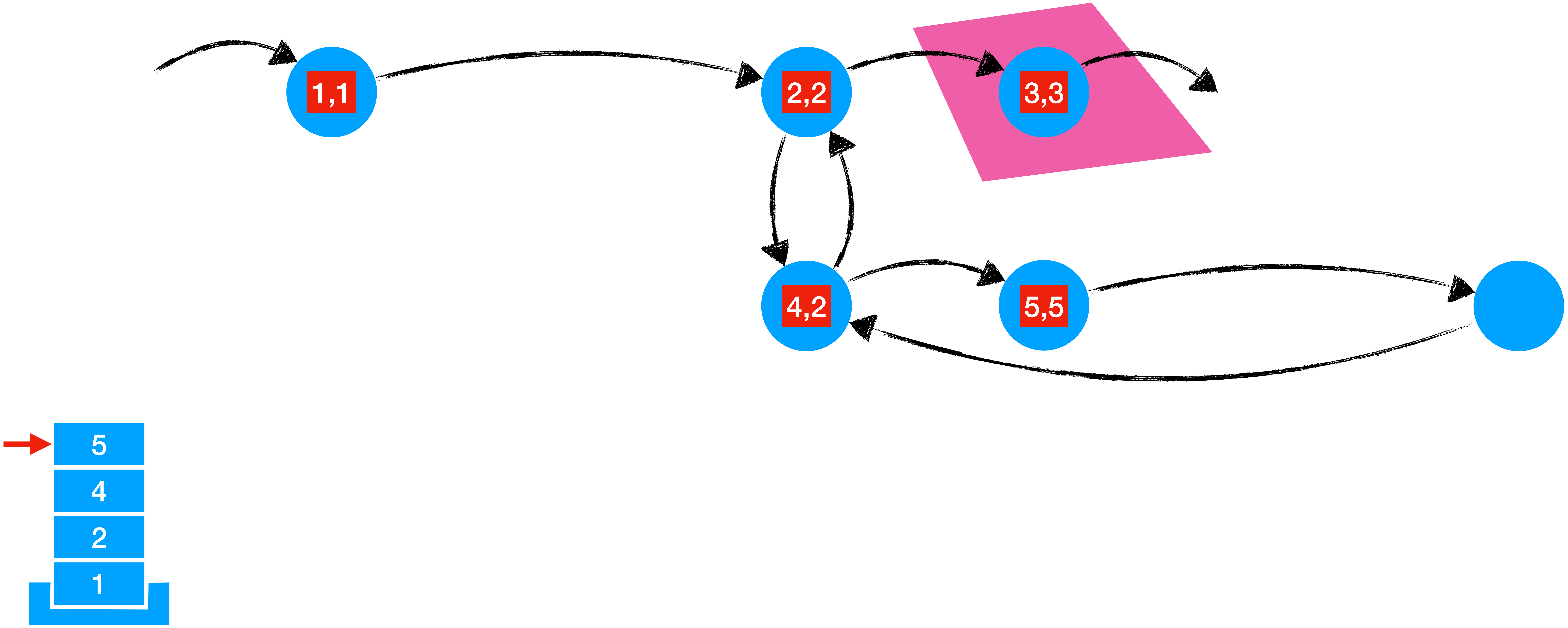

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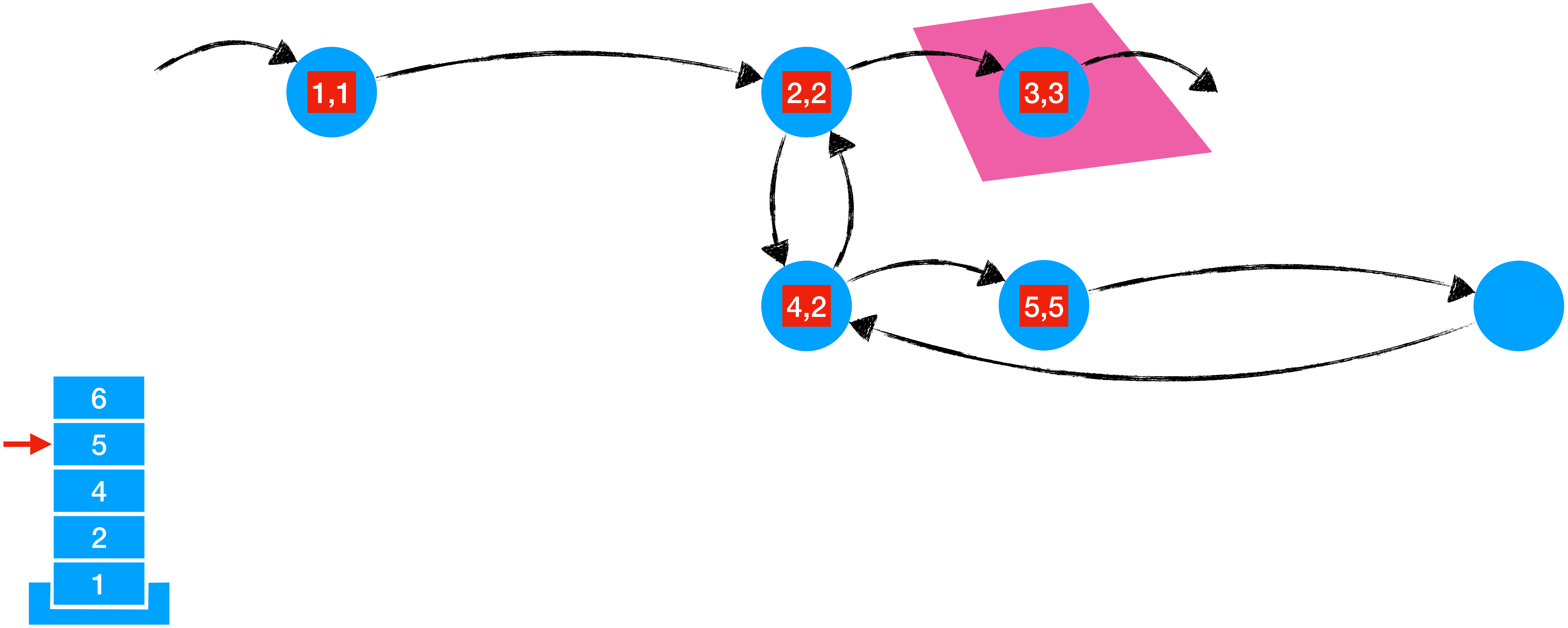

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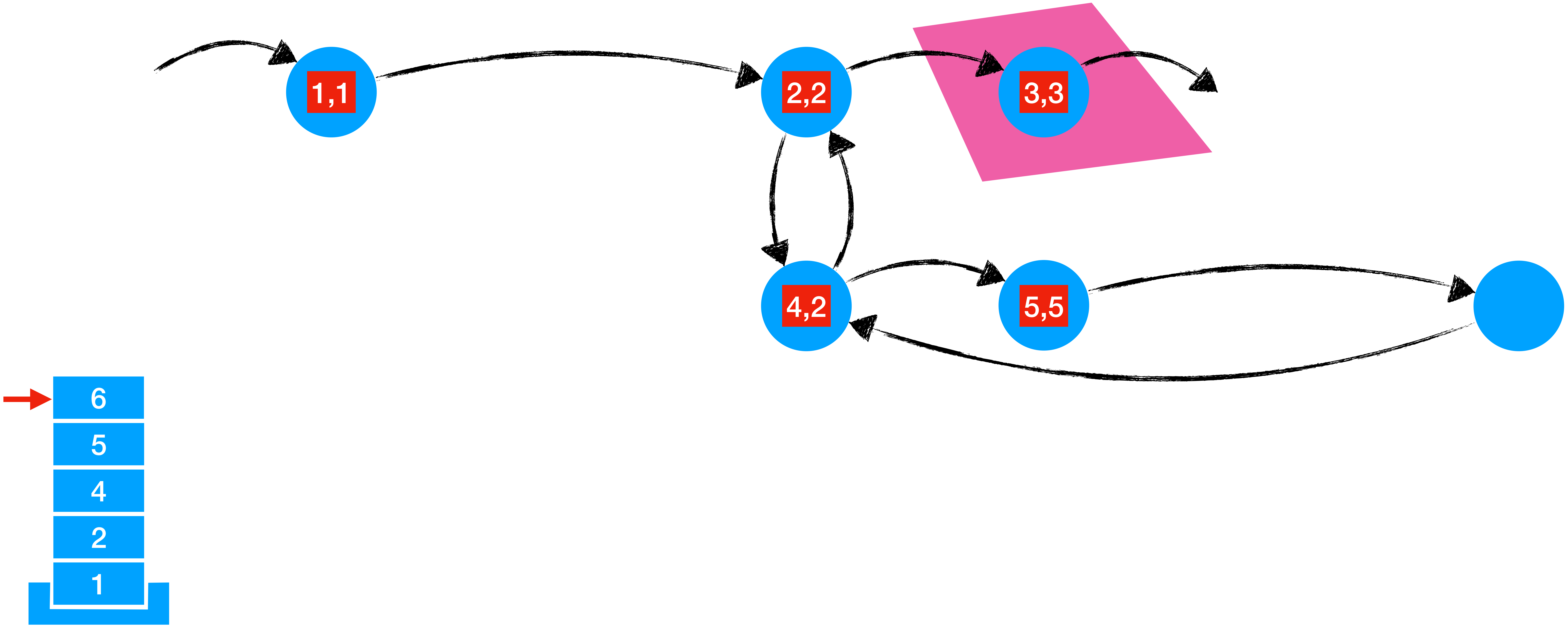

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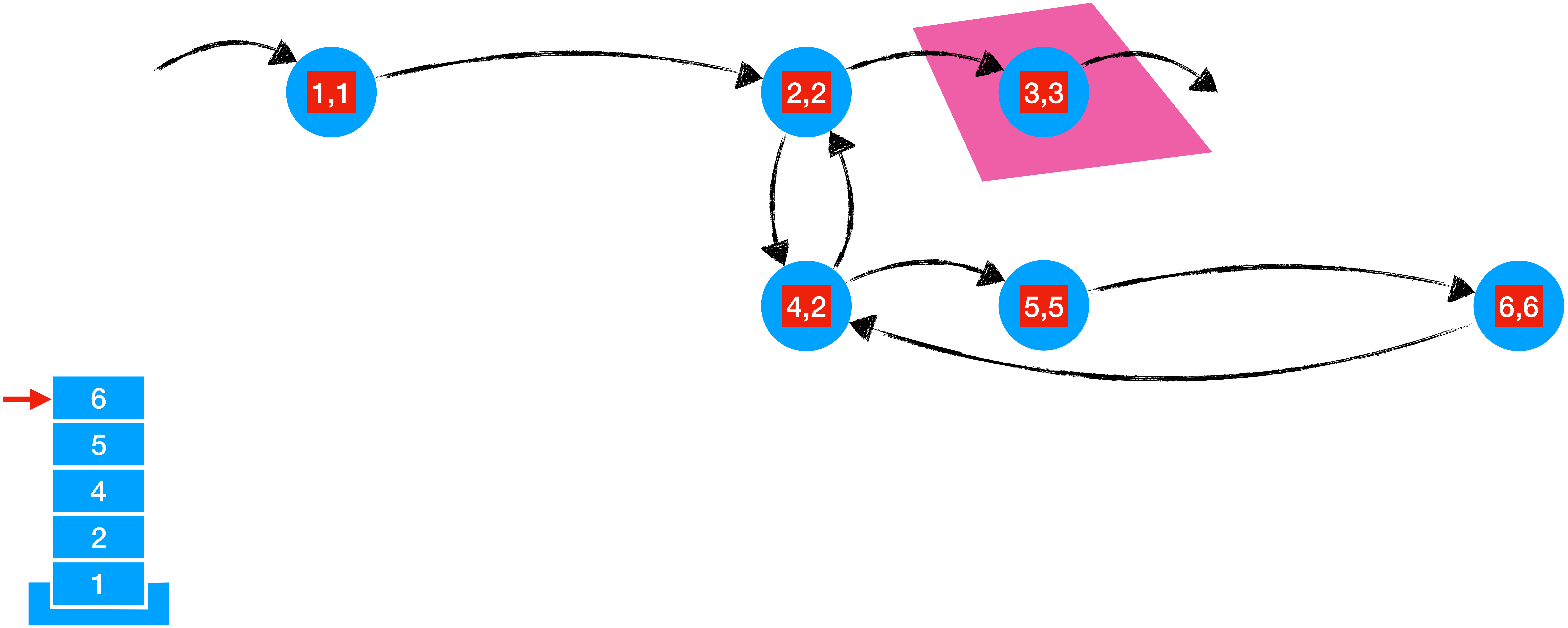

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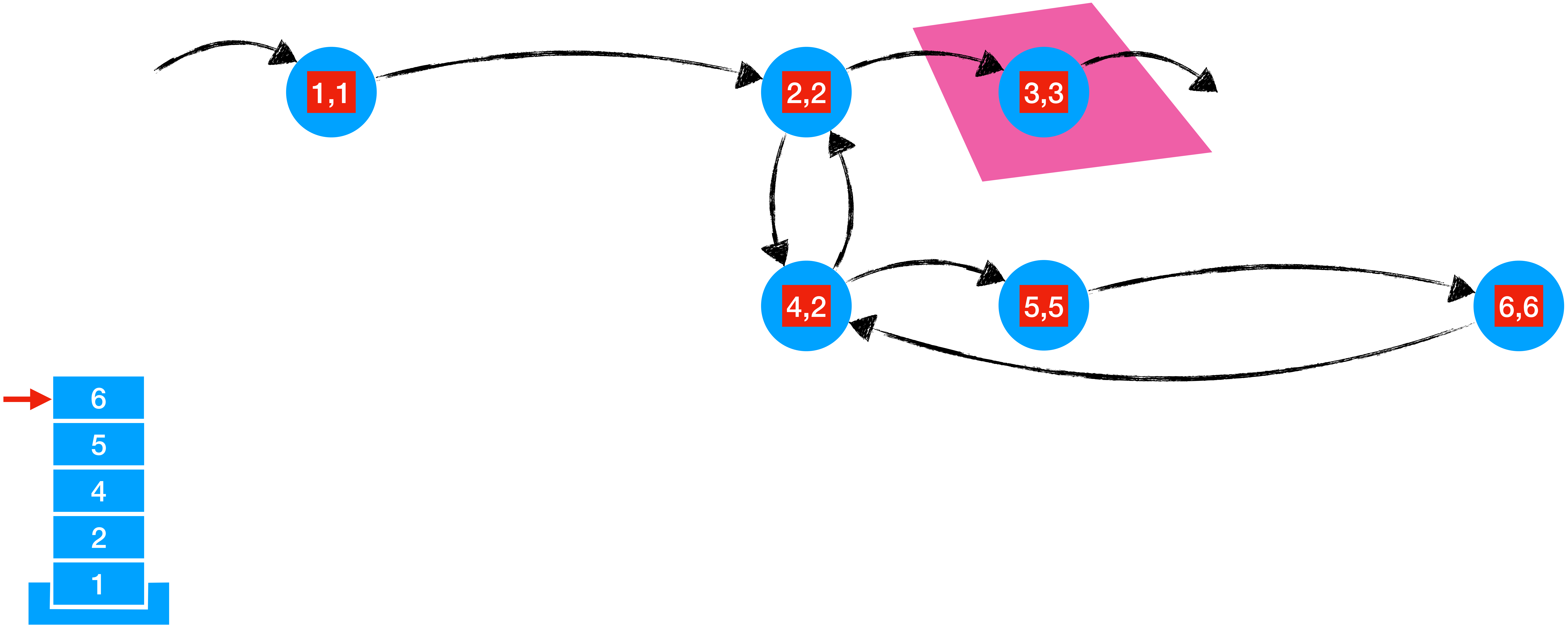

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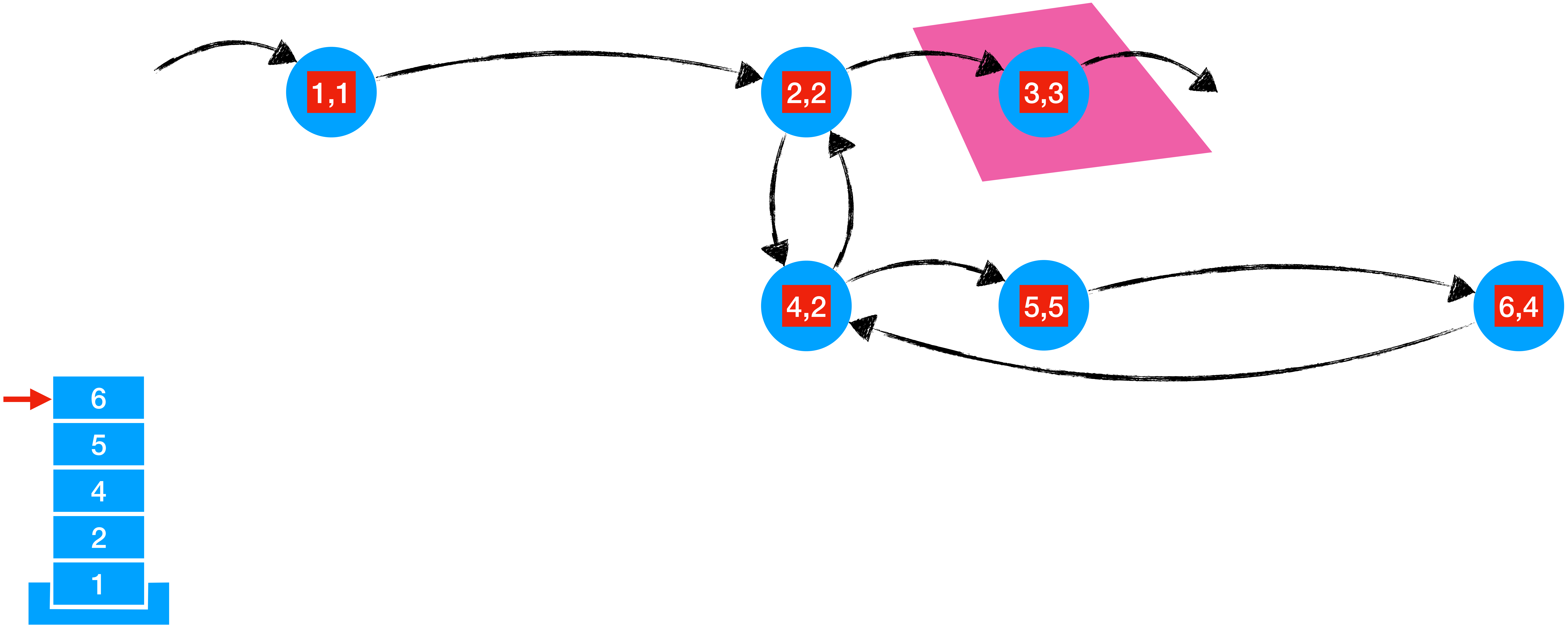

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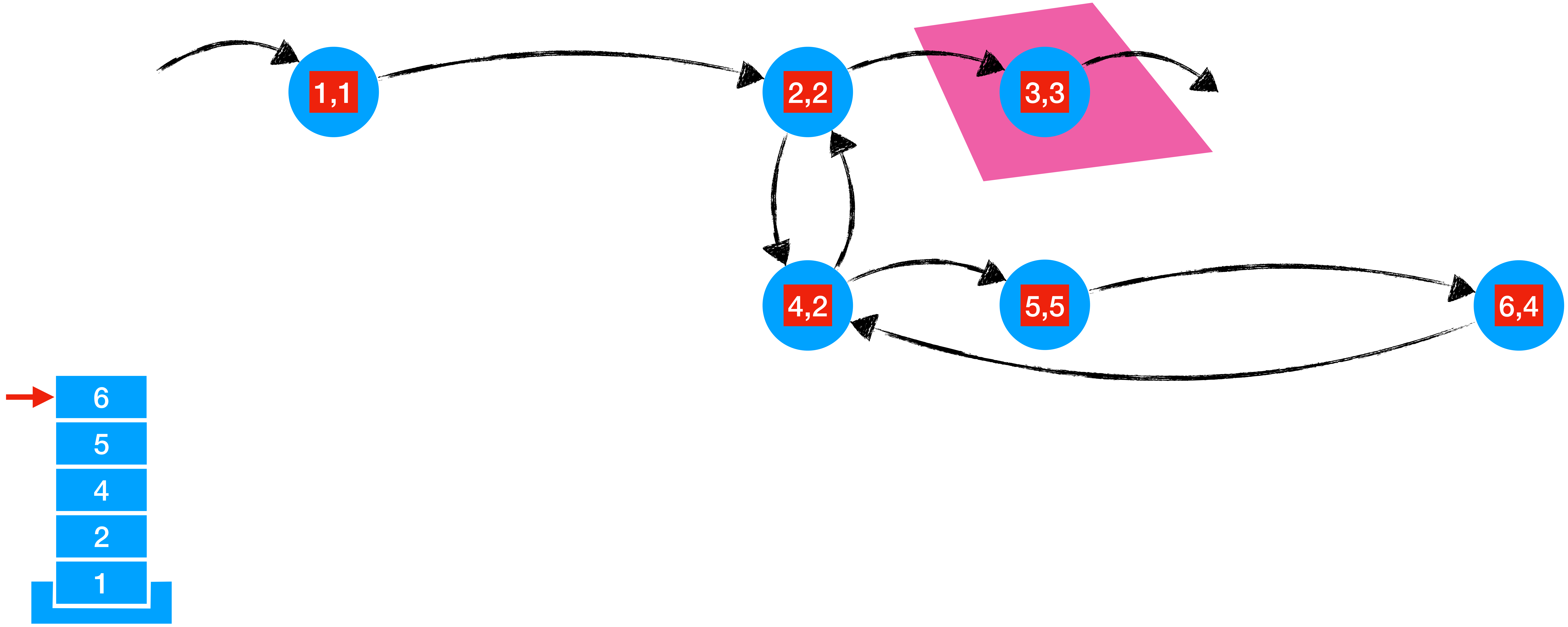

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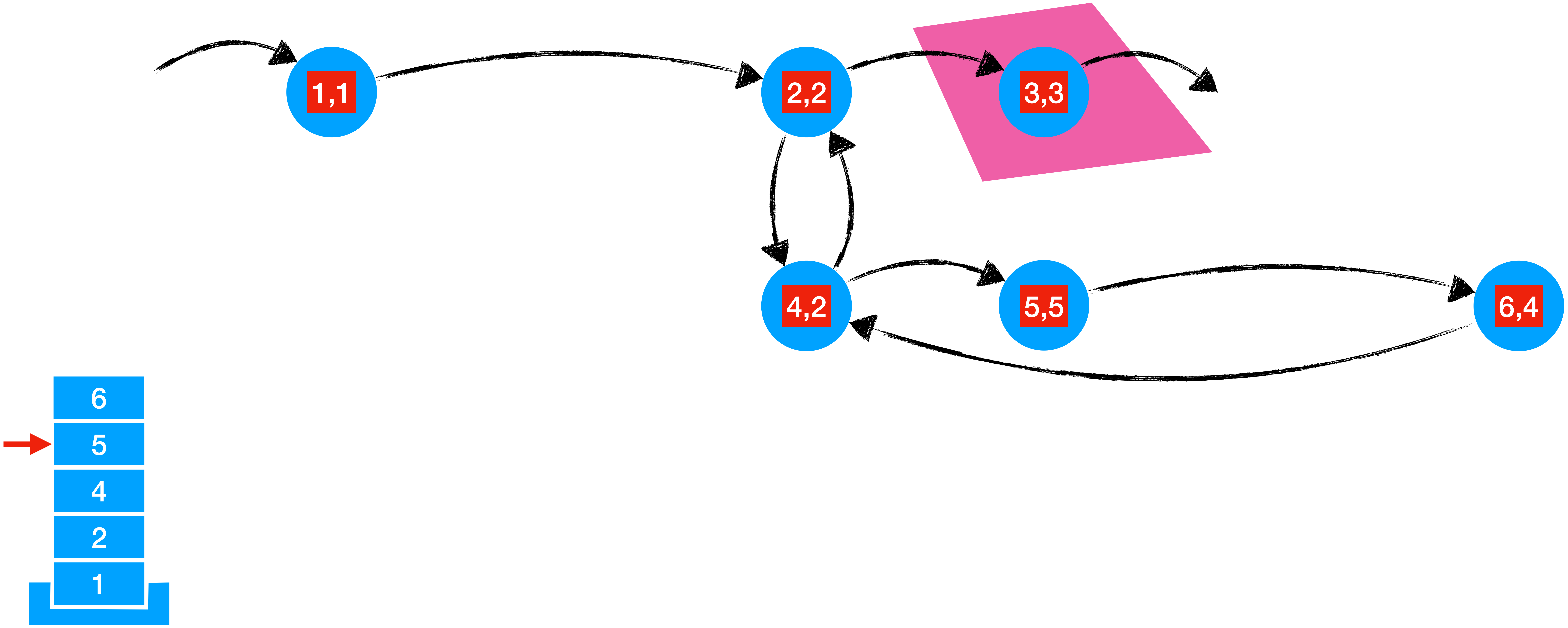

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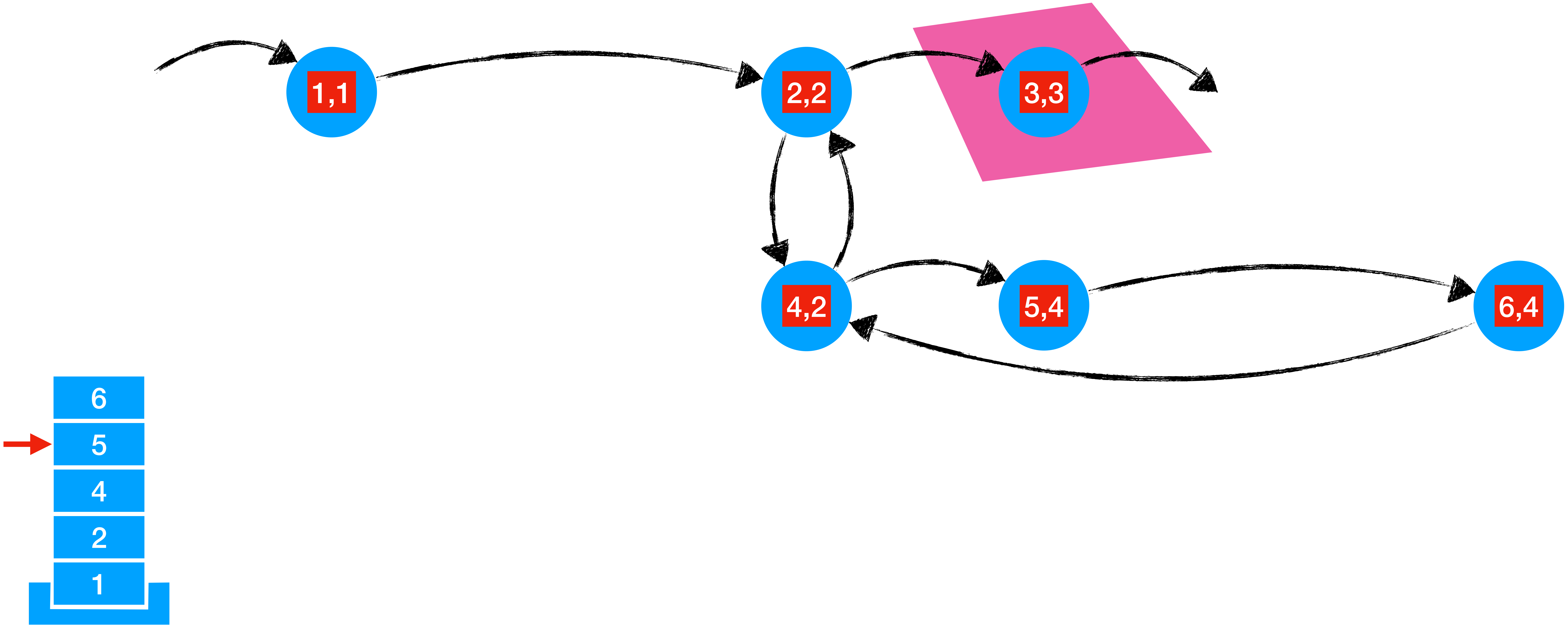

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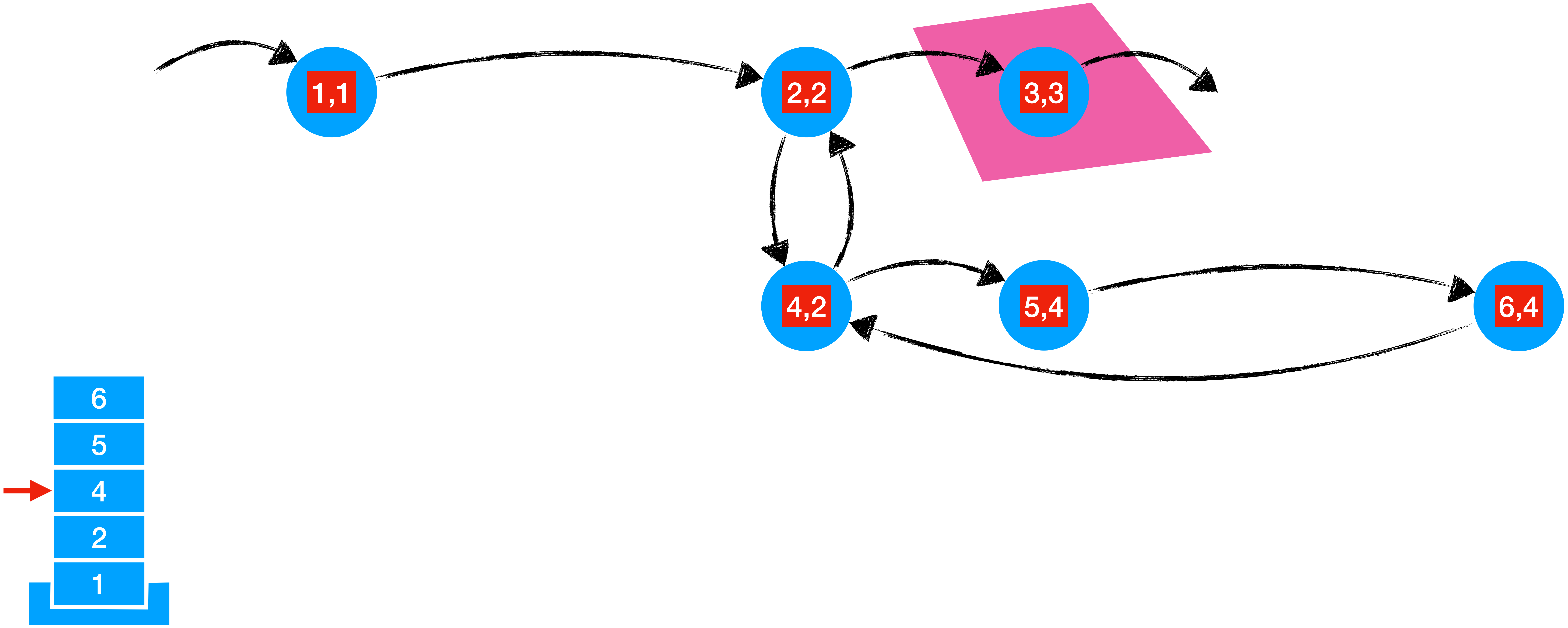

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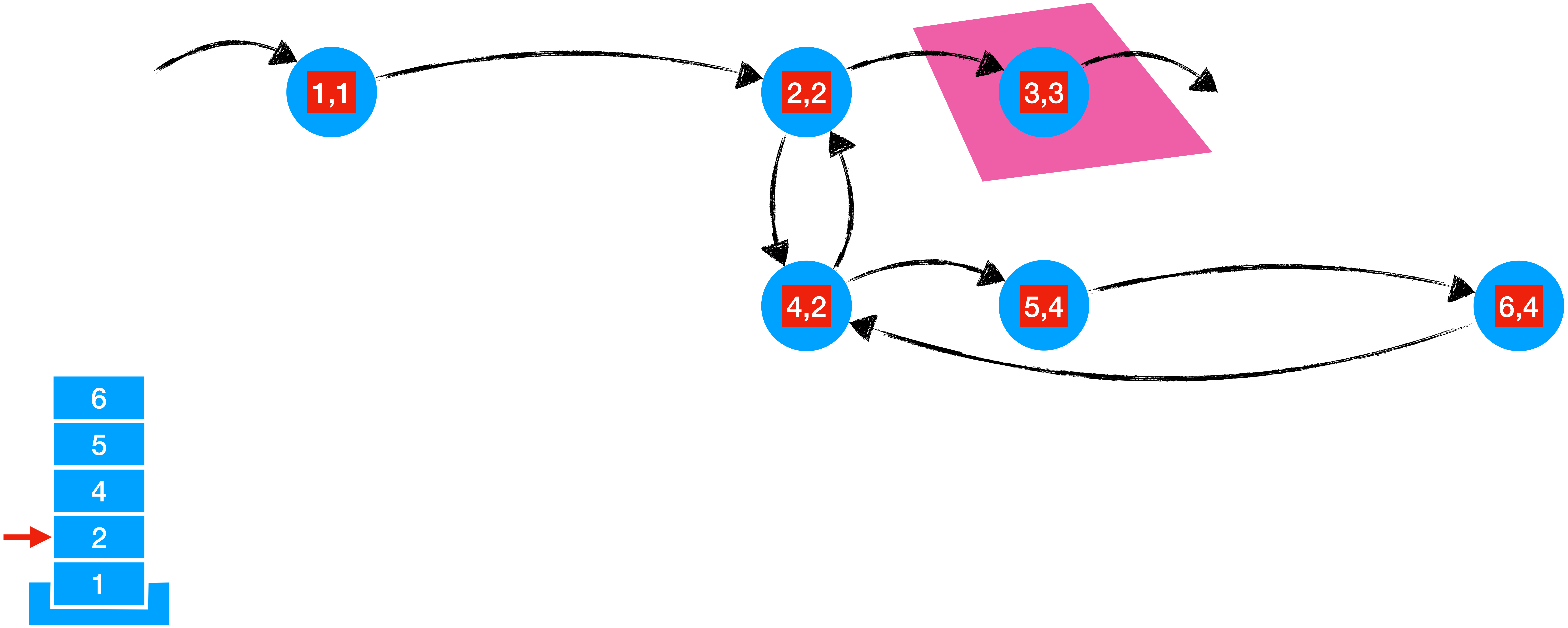

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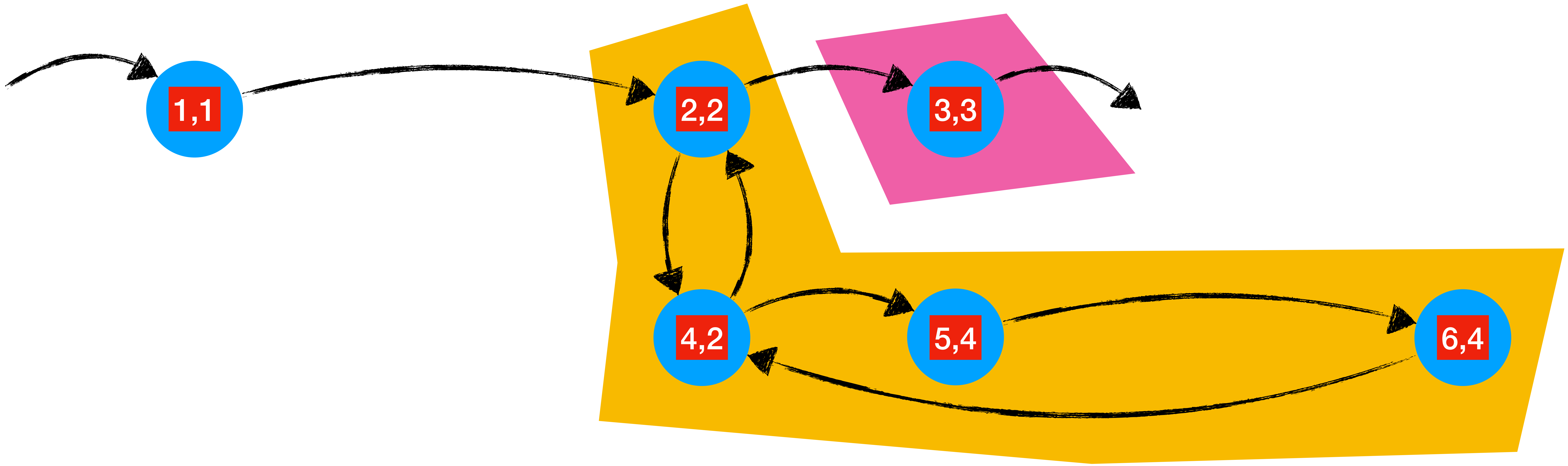

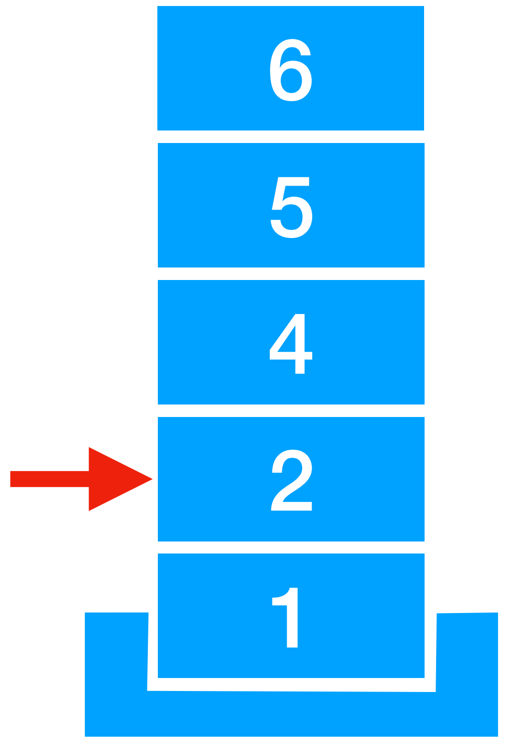

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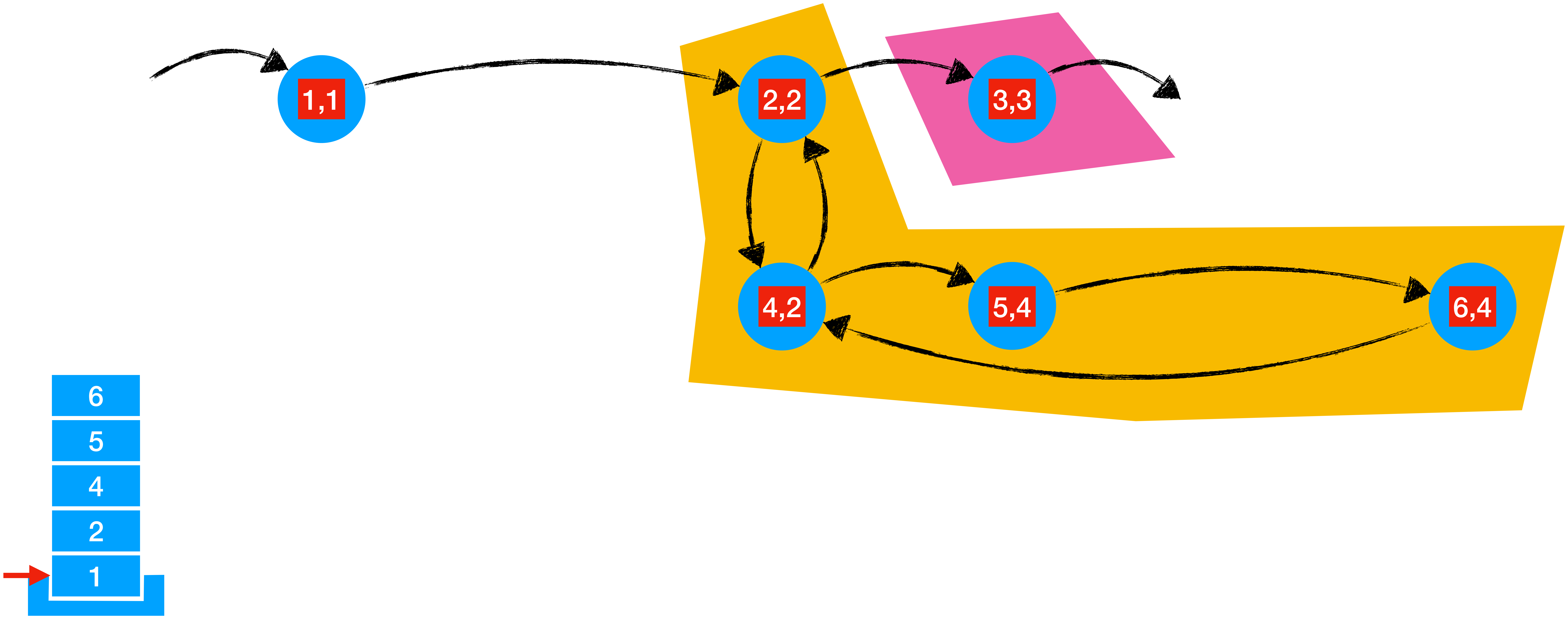

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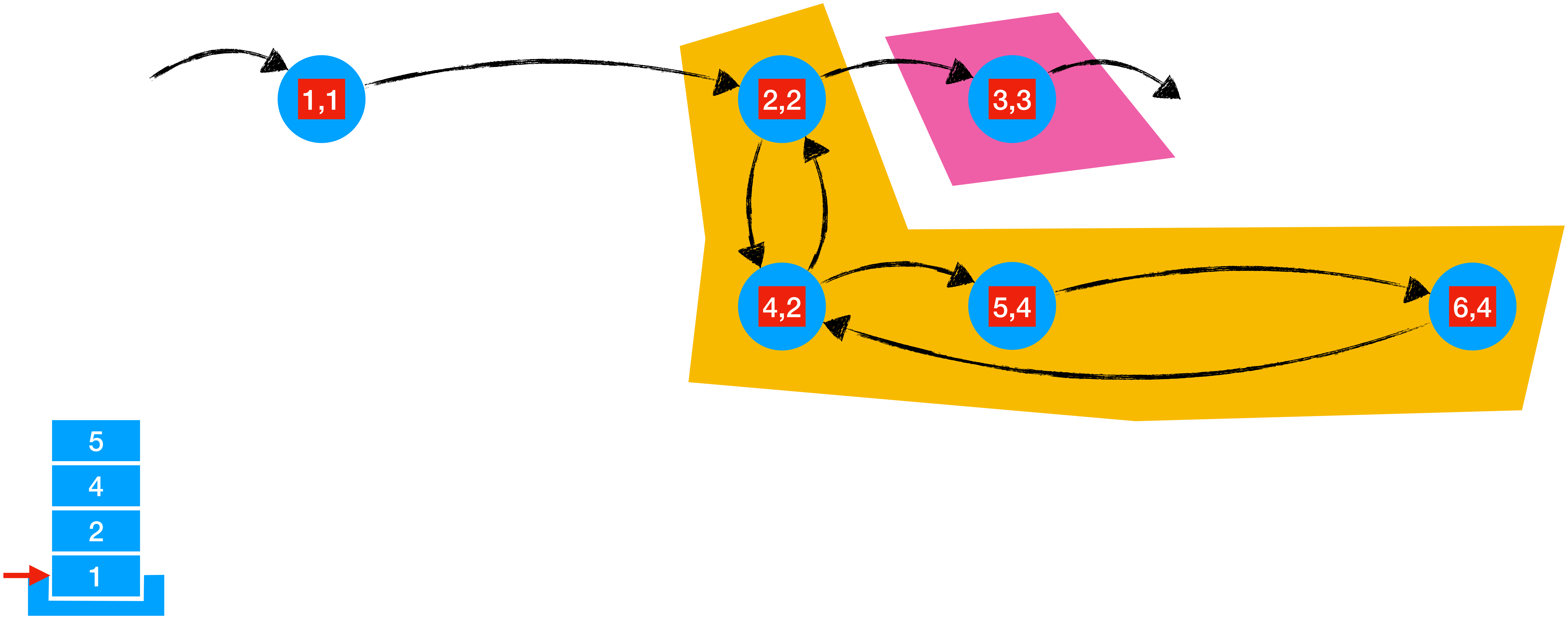

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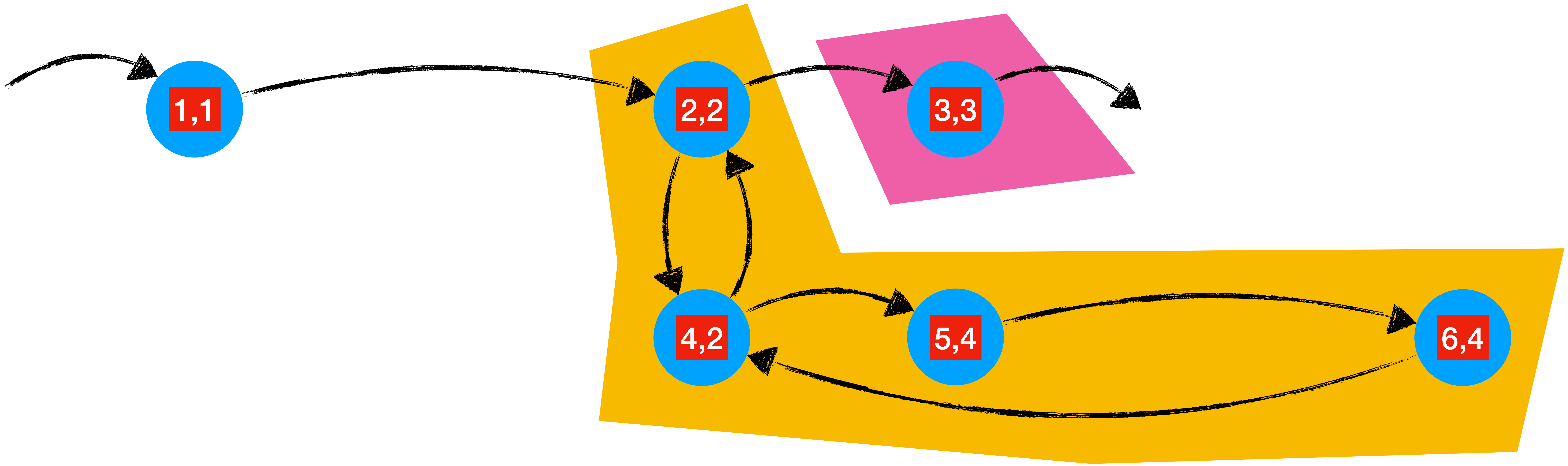

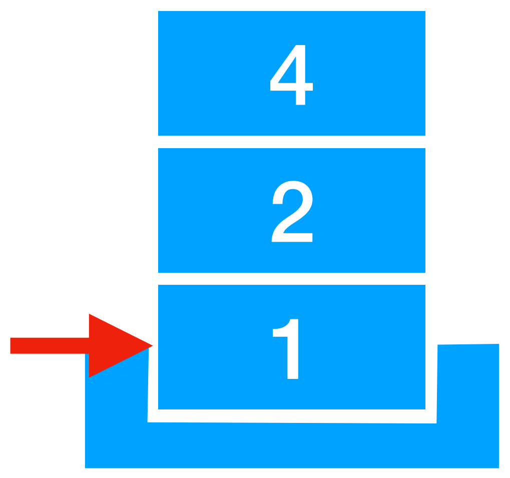

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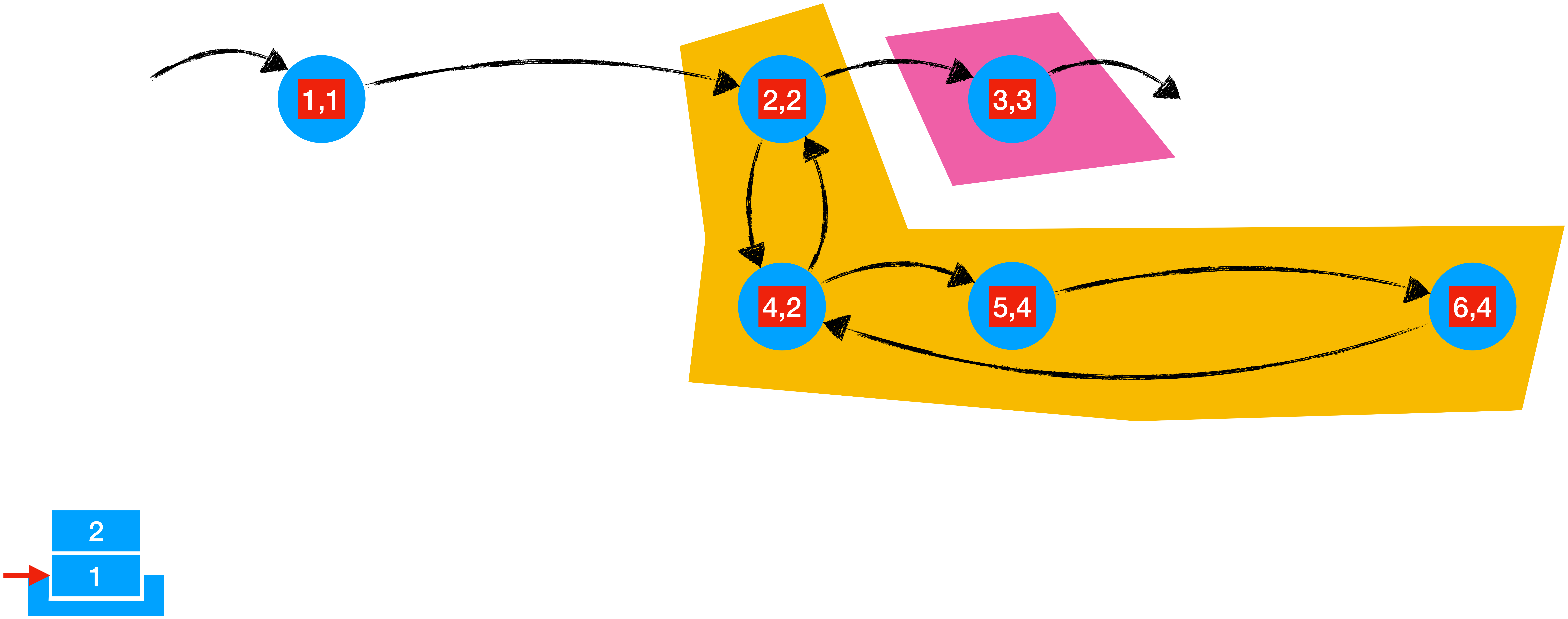

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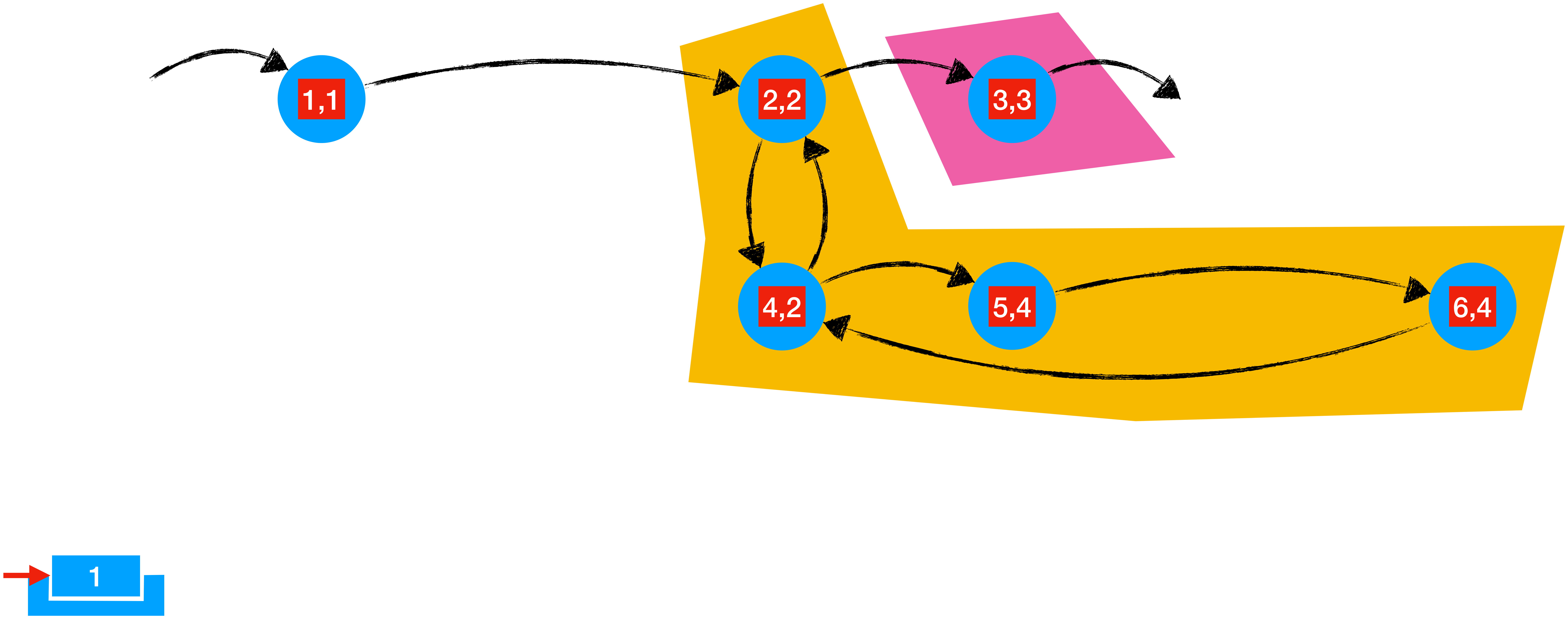

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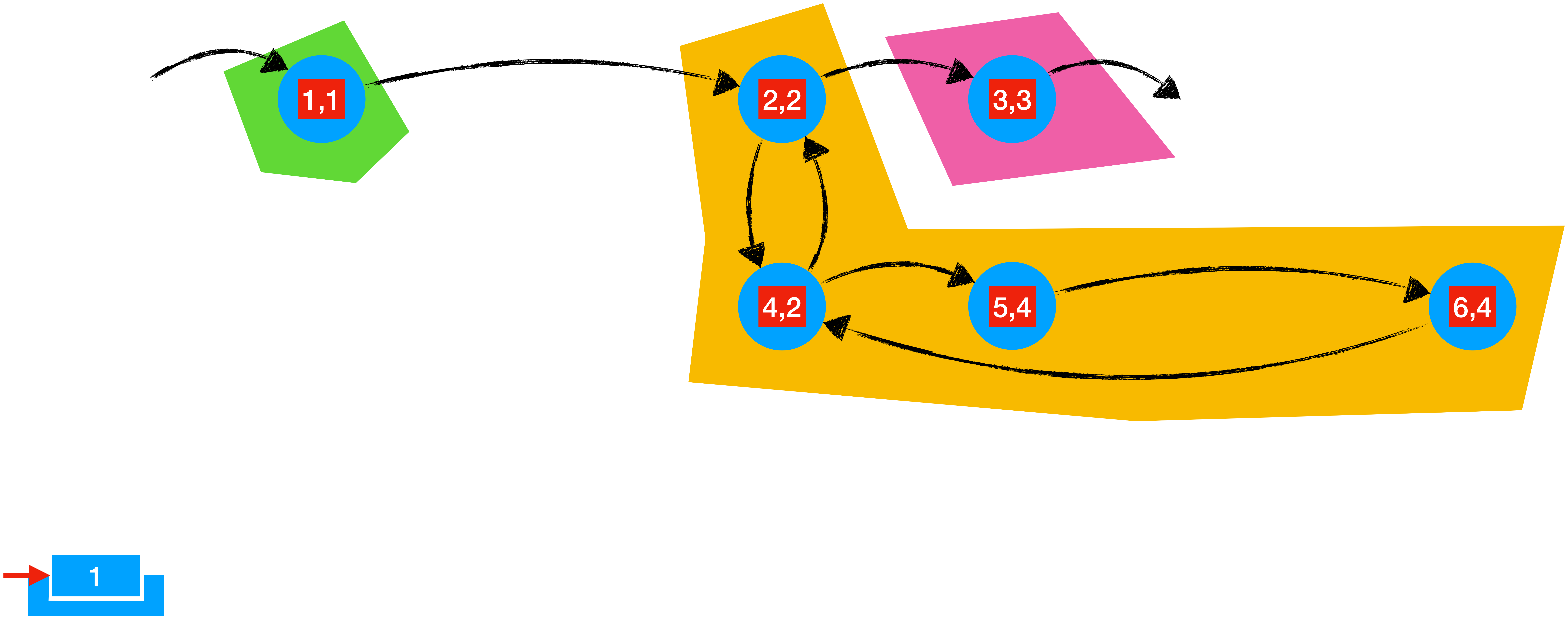

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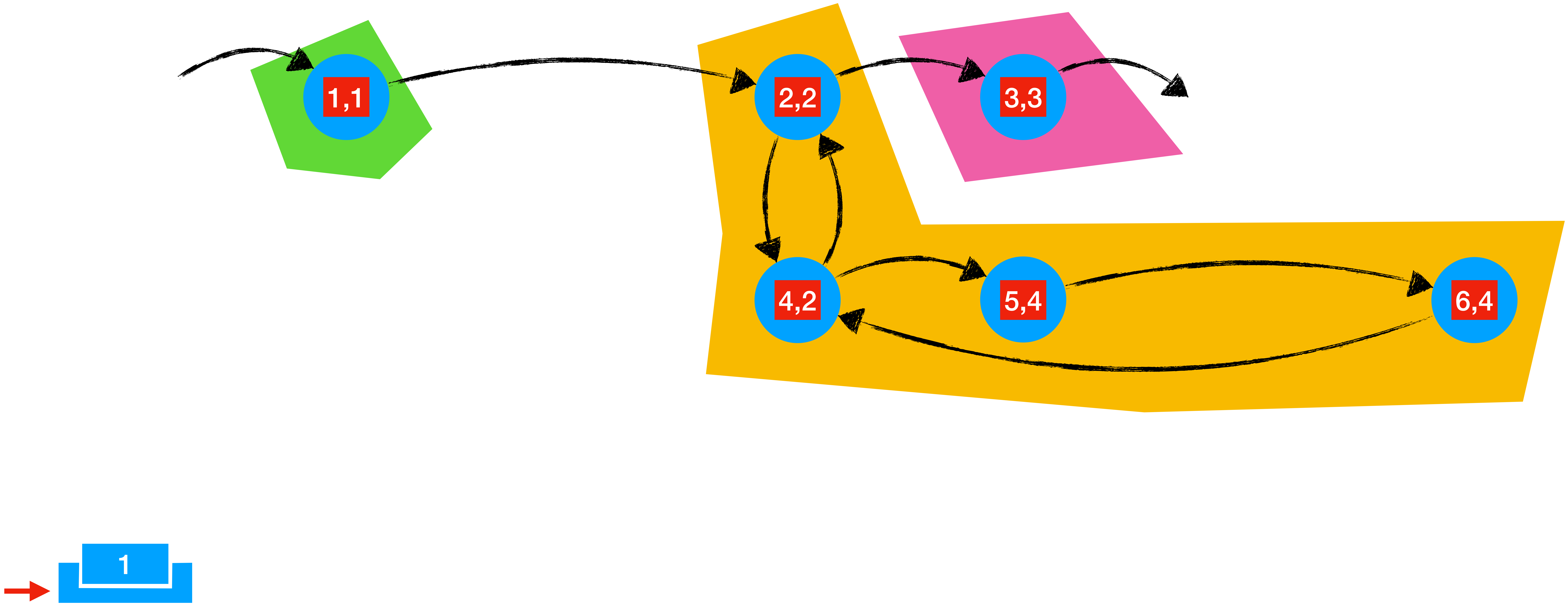

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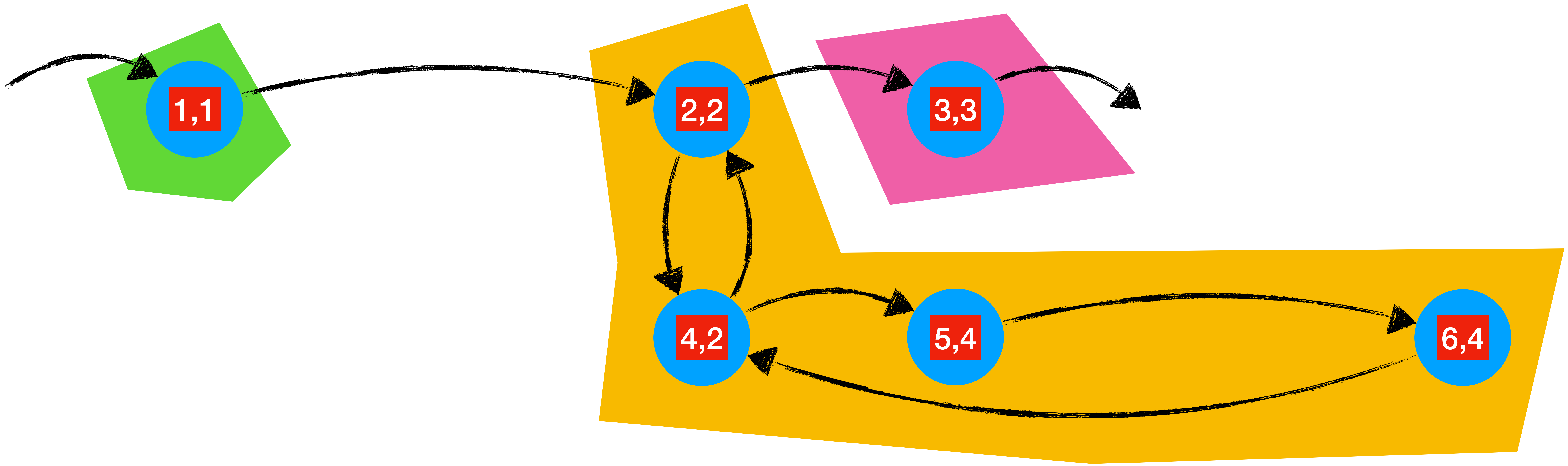
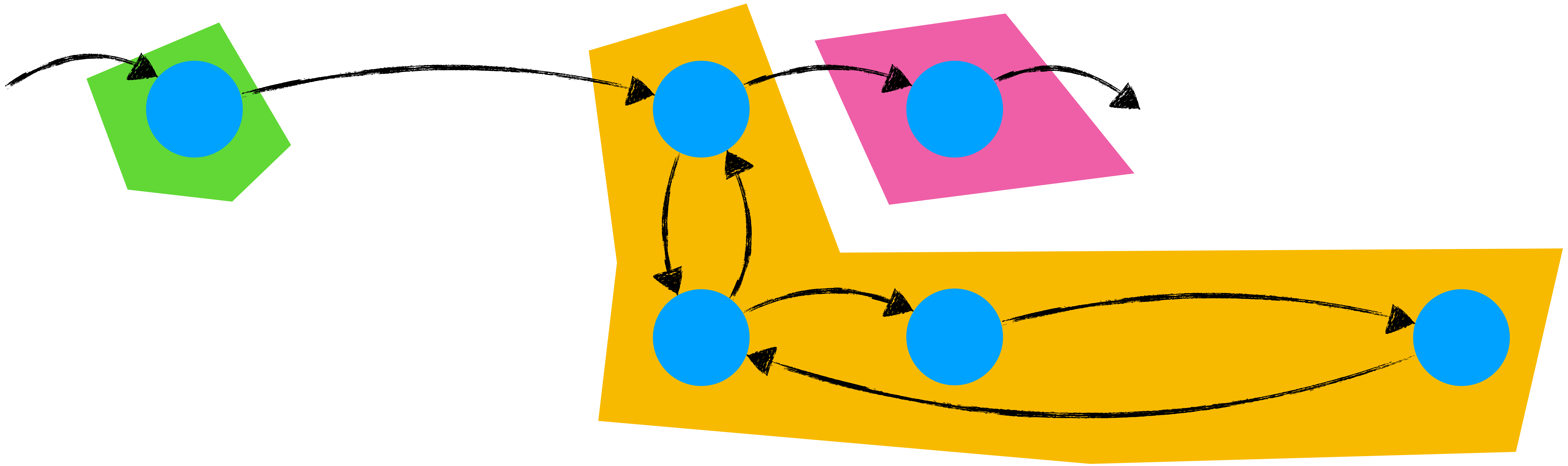


Reverse postorder in SCC



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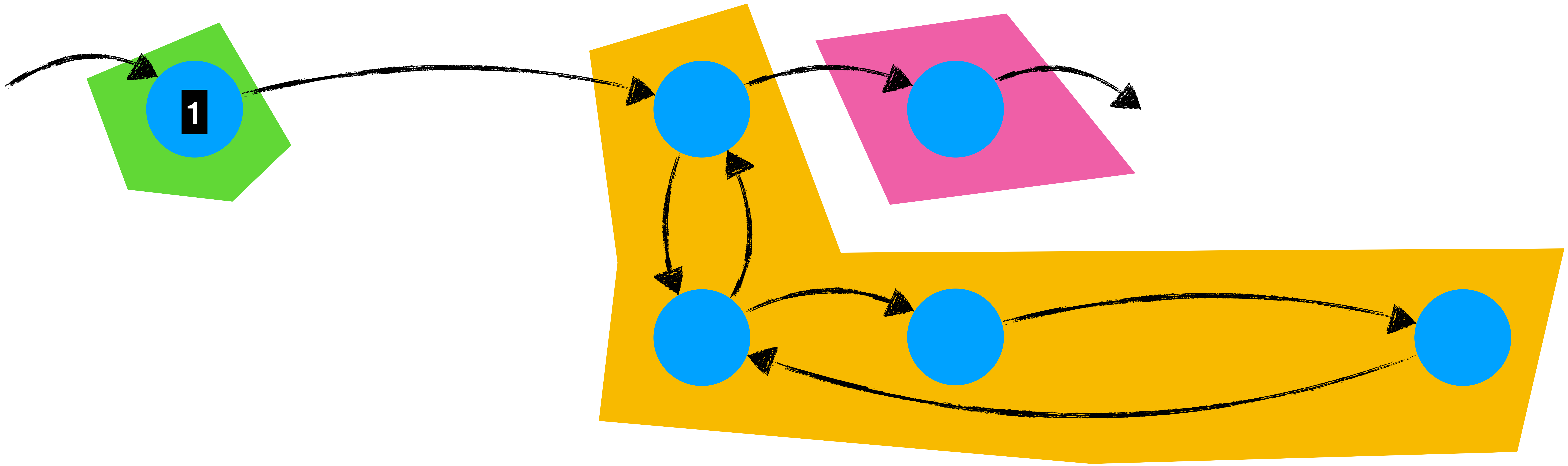

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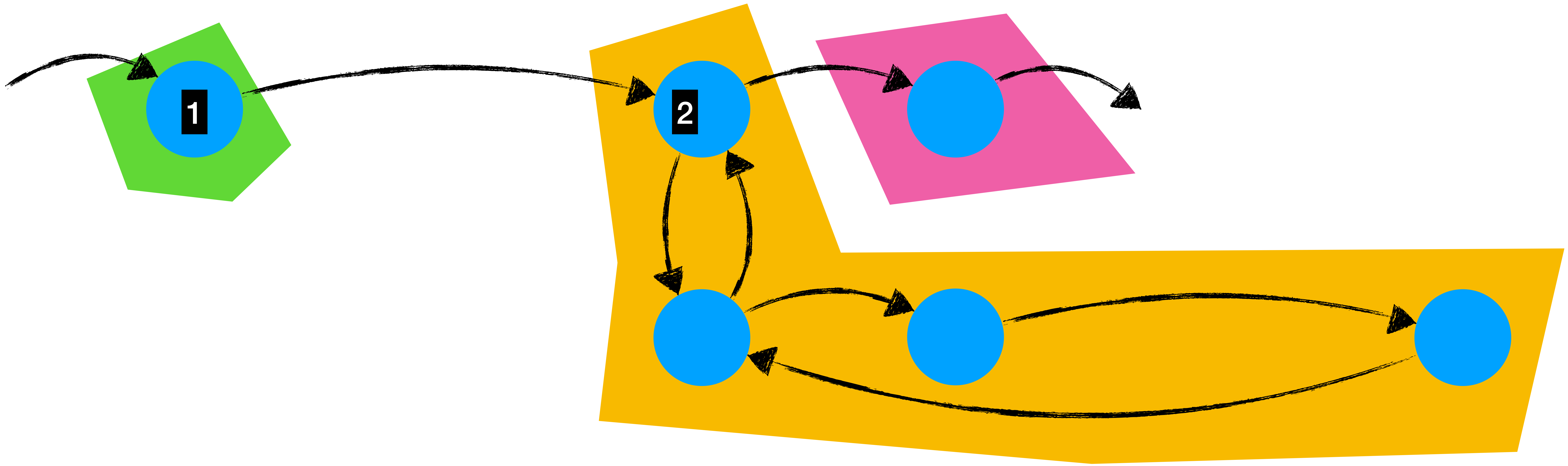

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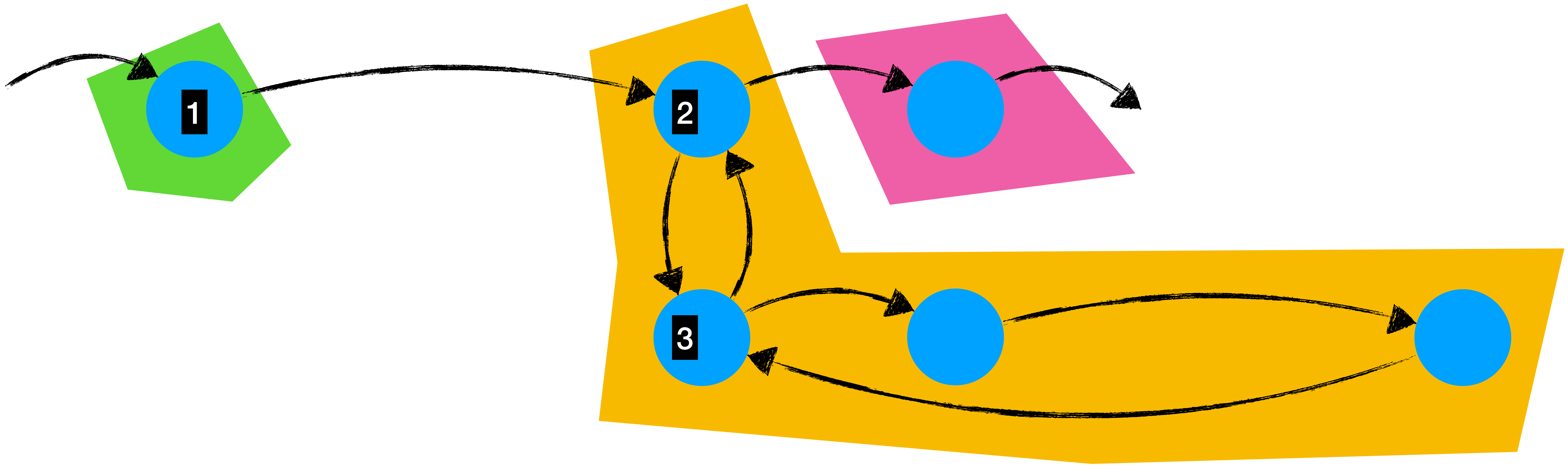

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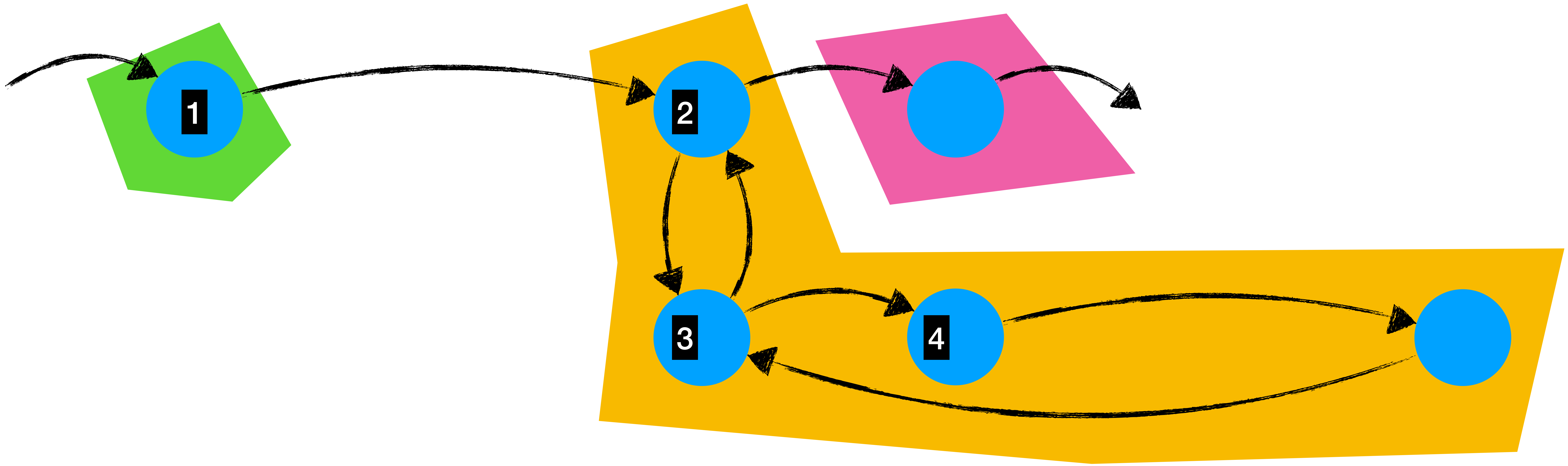

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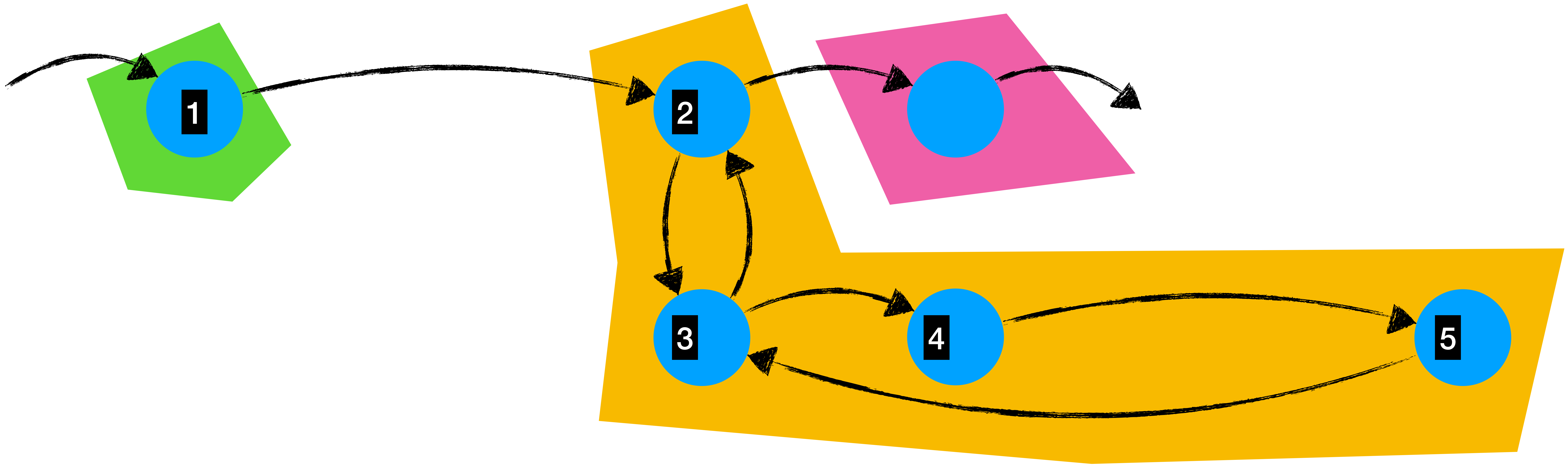

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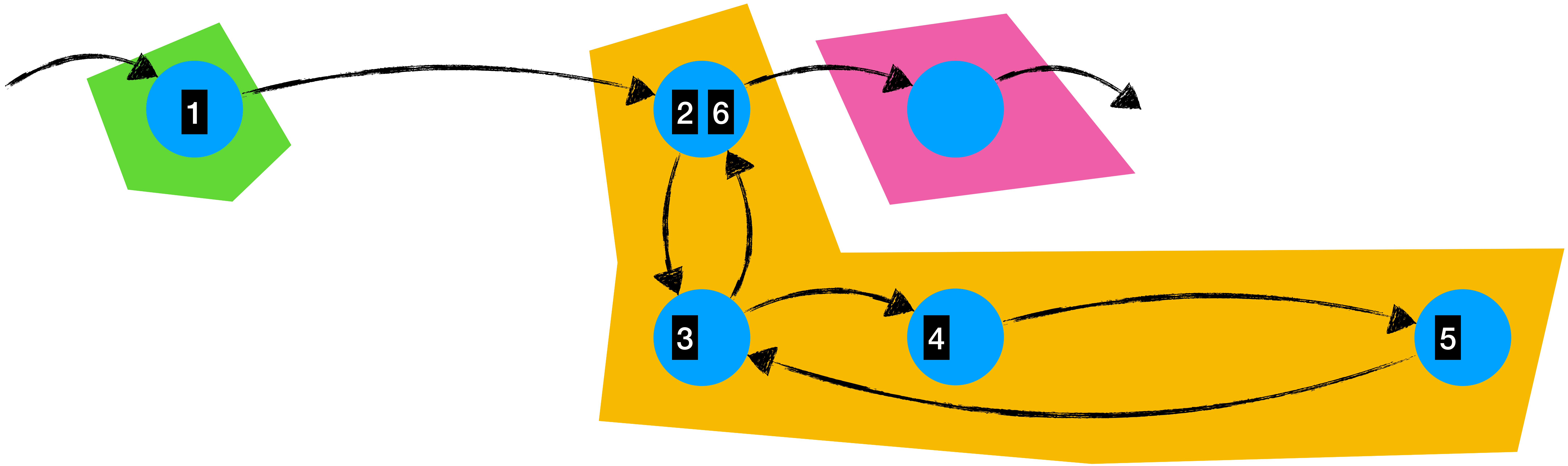

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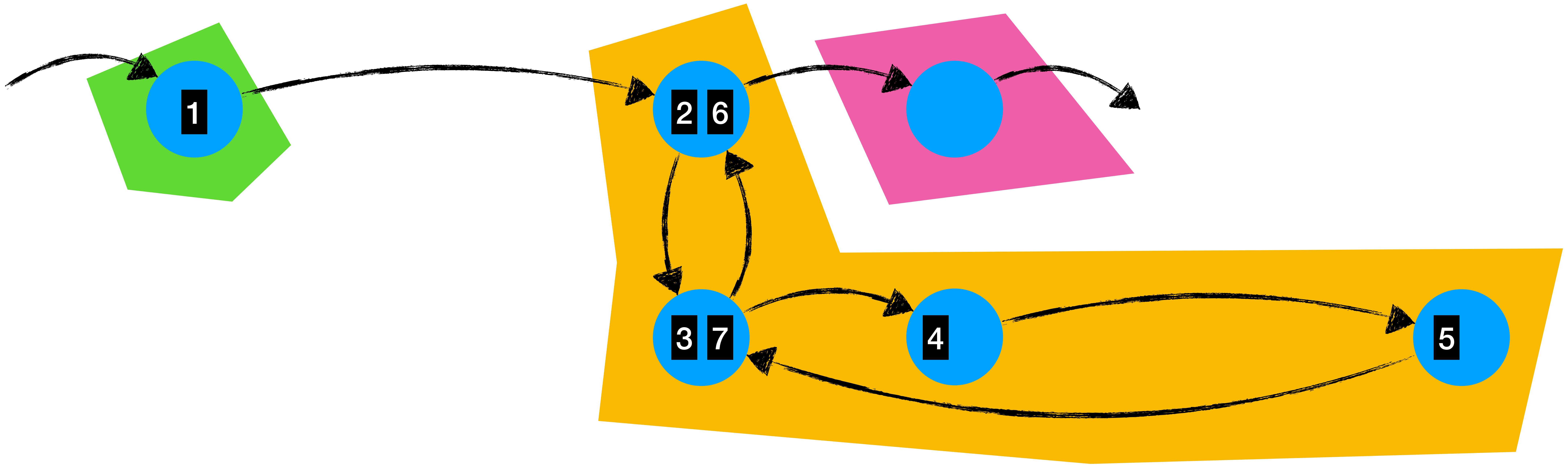

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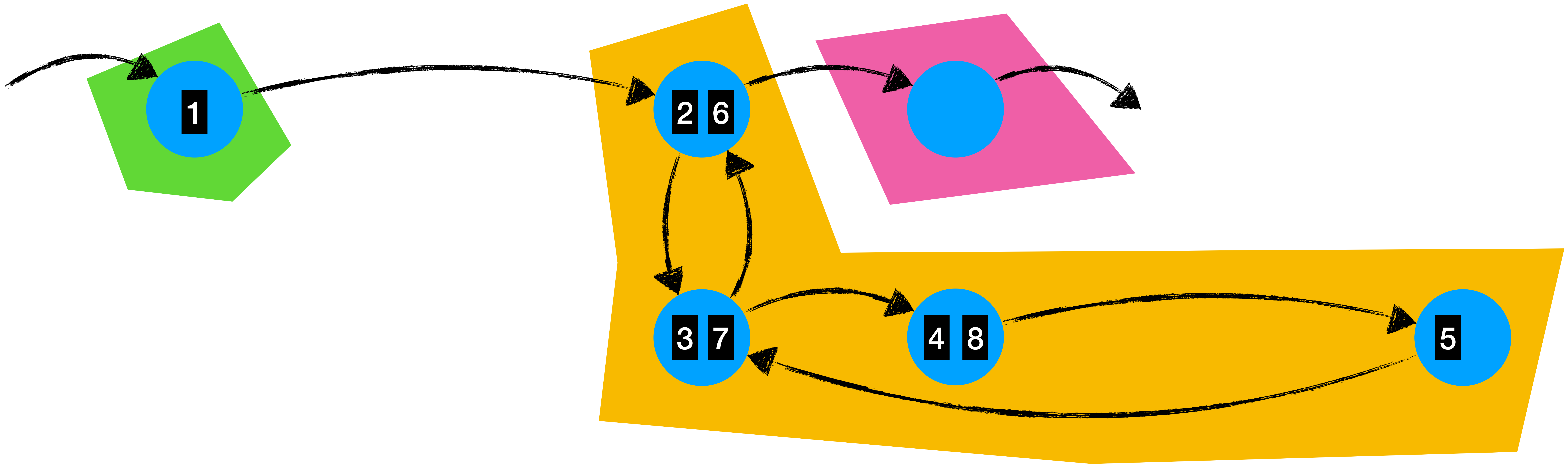

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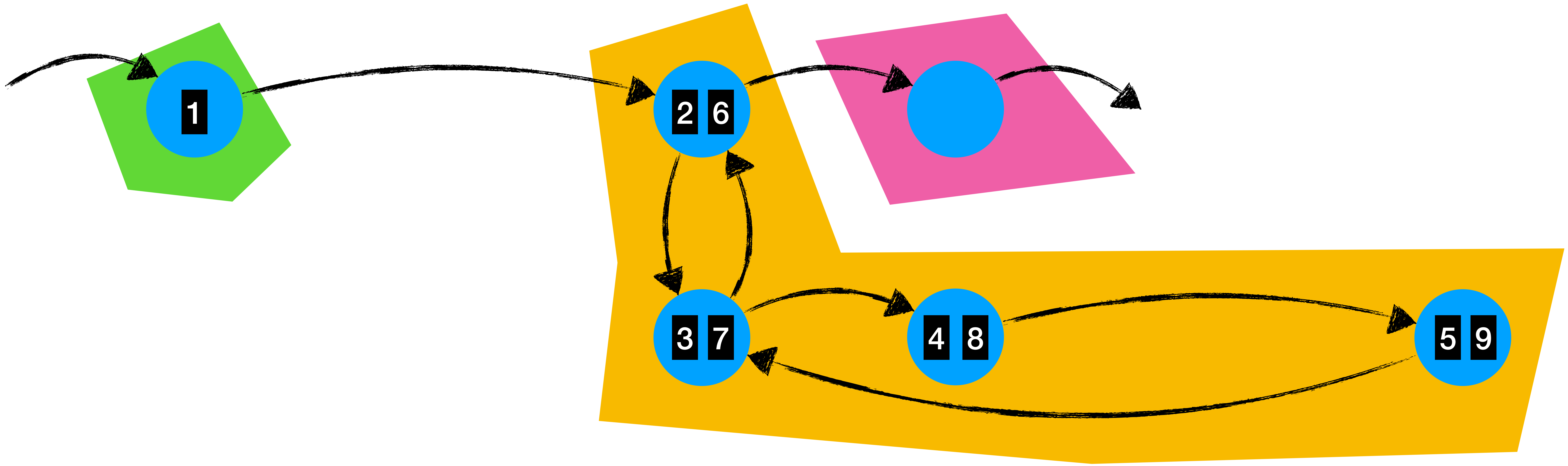

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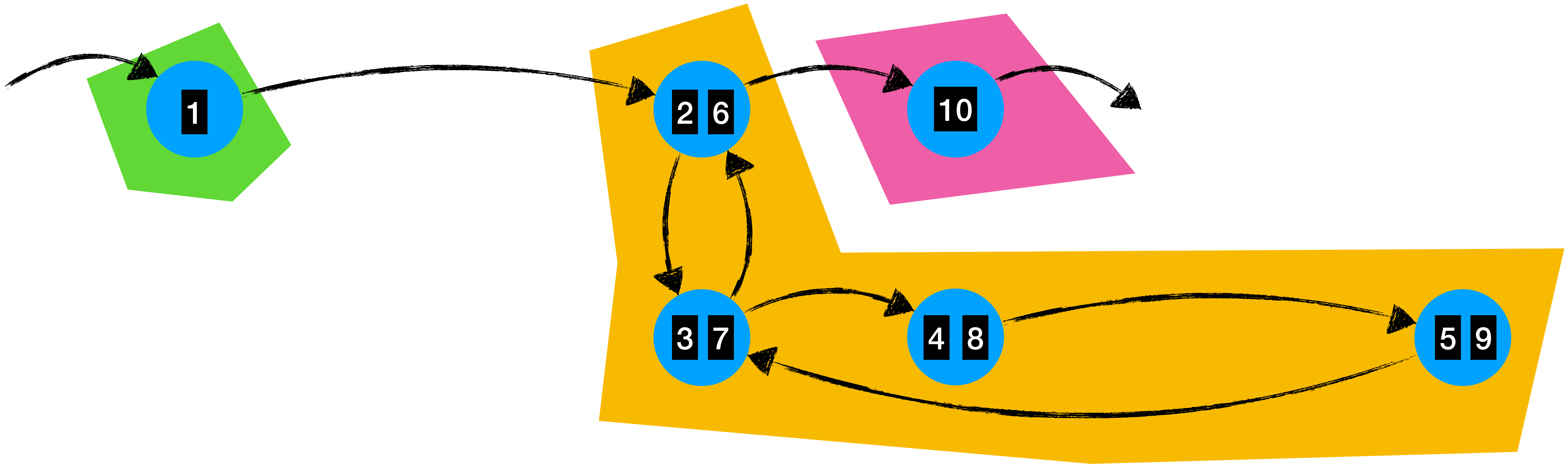

Conclusion

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- Worklist algorithm with optimisations:
  - SCCs, reverse post-order within SCC, CFG filtering